3-D Seismic Imaging for the 21st Century: Common Azimuth Wave-Equation Migration and MVA

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Summary

We present examples of practical 3-D wave-equation imaging case histories, demonstrating that the long-sought-after goal of accurate imaging has been attained. It is now possible to perform 3-D prestack wave-equation migration for the same computational cost as Kirchhoff migration, and at 50 times less cost than 3-D shot profile migration. Not only is it now possible to image 3-D seismic data with an algorithm that more accurately honors wave propagation in the earth, but it is also practical to use the imaging algorithm as a basis for velocity model building.

Common Azimuth vs. Kirchhoff and Shot Profile

Three-dimensional prestack imaging has been dominated by Kirchhoff integral equation methods because the latter have been the only practical methods; however, integral methods have their shortcomings, and a great deal of effort has been spent trying to overcome Kirchhoff's limitations. Recently, Biondi and Palacharla (1996) had the insight that for most marine acquisition geometries, the wave equation can be implemented more directly than in Kirchhoff by applying a stationary phase approximation to the differential form of the double square root (DSR) equation. The physical insight that marine data are acquired over a limited azimuth range, combined with the mathematics of acoustic wave propagation lead us to the practical implementation of wave-equation migration known as the common azimuth method (COMAZ).

Critics correctly point out that the common azimuth approach is an approximation; however, so is Kirchhoff migration. Kirchhoff migration covers any and all azimuths, but the method makes large approximations in computing the Green's functions. In principle, Kirchhoff migration can be as accurate a solution to the wave-equation as any finite difference or phase shift downward continuation, but in practice a tremendous number of approximations are made in Kirchhoff: The Green's functions are approximated by traveltime tables that asymptotically approach the full wavefield. Even multivalued or energetic traveltime calculation approaches are very subjective and prone to numerous assumptions, and can never capture all the energy, modes, and phase shifts that can be captured by a fully recursive migration (caustics, triplications, focusing and defocusing, etc.). Errors are introduced when sampling the traveltimes, by several approximations. First the traveltimes are computed on a sparse grid representing the shot locations, and then interpolated for each actual shot location. Second, the traveltimes computed on a sparse grid are interpolated for the imaging grid. Amplitudes are approximated by solving the transport equation or by ray tracing. While the Kirchhoff integral method accomodates irregular and uneven sampling, numerous considerations and approximations must be made in terms of trace weighting and integration rules – these are prone to a great number of assumptions and errors. Antialiasing of Kirchhoff integral operators is another area that gives rise to numerous assumptions and approximations. Proper antialiasing is dependent on dip spectra of the data, bandwidth, and propagation velocity, which are not always completely and accurately specified. In contrast, common azimuth migration doesn't have the numerous parameters that have to be set in Kirchhoff, and is therefore less prone to operator error in job setup.

Both shot profile and common azimuth wave equation migration are recursive wave equation migration methods. Shot profile migration handles a wide range of azimuths, while common azimuth migration only handles a narrow range (or a wider range in narrow azimuth). The disadvantage of shot profile migration is that it requires fifty times more computations than common azimuth migration. Just as Kirchhoff migration can be made arbitrarily fast by severely limiting aperture, shot profile migration can be made fast by limiting the aperture into which the shot is downward continued. Therefore, any speed claims that can be made for shot profile, hold fifty times over for common azimuth.

Wave-equation based migrations algorithms that downward extrapolate the 3-D prestack wavefield have been recently shown to produce better imaging results than Kirchhoff migration in many synthetic and real data cases. A list of wave-equation migration properties (Popovici, 2000) that contribute to improved imaging results is given below:

- Wave-equation methods are potentially more accurate and robust because they are based on the full wave equation and not on an asymptotic solution based on ray theory.
- Wave-equation methods handle multipathing naturally, while Kirchhoff methods do not.
- Focusing and defocusing effects of the velocity variations are correctly modeled by wave-equation methods.
- Antialiasing is handled implicitly.
- There is no need for traveltime interpolation, analysis of traveltimes grid error or traveltime shot spacing, or massive disk storage of traveltime tables.
- The ability to generate Common Image Gathers for velocity analysis and residual NMO before stack.
- Faster than Kirchhoff for full volume.
- Accurate amplitudes, consistent with the wave-equation.
- Simple user interface, fewer parameters to set, easier to use by data processing personnel.

Figure 1 shows a comparison of imaging results using 3-D prestack depth migration Kirchhoff and COMAZ. The wave-equation migration result shows better resolution, better accuracy and better amplitude characterization of the salt/sediment interface.

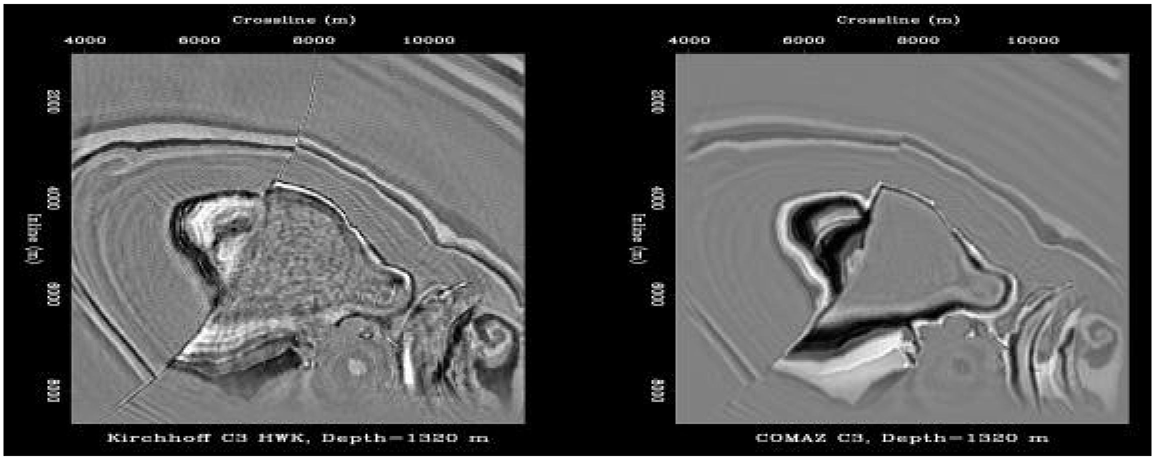

Figure 1. Depth slice through the SEG/EAGE C3 Salt model using 3-D prestack Kirchhoff and COMAZ. Notice the better resolution and accurate salt flanks in the COMAZ wave-equation migration result (right).

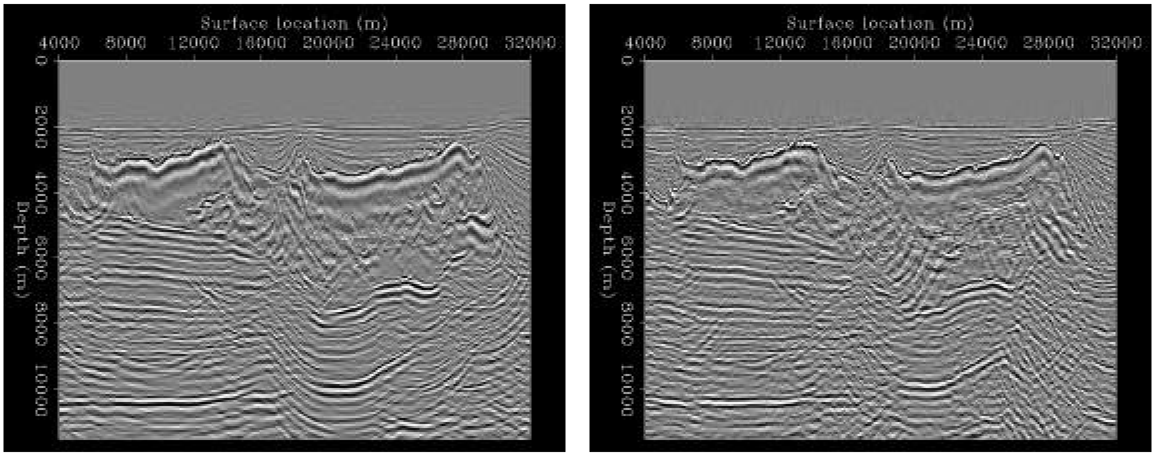

Figure 2. Left: Kirchhoff depth migration. Right: Common Azimuth wave-equation migration. Notice the imaging improvements under salt, less migration artifacts, better horizon continuity. Deep USA data courtesy of Unocal.

An important feature of COMAZ, nonexistent in the industry until a few years ago, is the ability to generate Common Image Gathers (CIG) in either offset ray parameter or angle domain (Prucha et al., 1999). This development opens the possibility to perform migration velocity analysis (MVA) and iterate to improve both the velocity model and the final migrated image. In a later section, we explain in detail how we use CIGs for MVA, and the benefits of building a velocity model with the same migration that is used to perform the imaging.

3-D Wave-equation MVA

An important step in prestack depth migration is the velocity model building process. The most accurate way of reconstructing velocity models is prestack migration velocity analysis (MVA) based on common image gathers. The CIGs contain redundant structural information that is used to correct the initial velocity model. Furthermore, velocity model building is most accurate if updates are based on prestack gathers generated from the same imaging algorithm that is used for the final imaging step. It is therefore imperative that wave-equation migration imaging is intimately related to wave-equation MVA. For this reason, we perform MVA using angle-domain common image gathers (ACIG) generated by COMAZ. These output image gathers are sorted in either reflection angle or ray parameter, as shown in Figure 3.

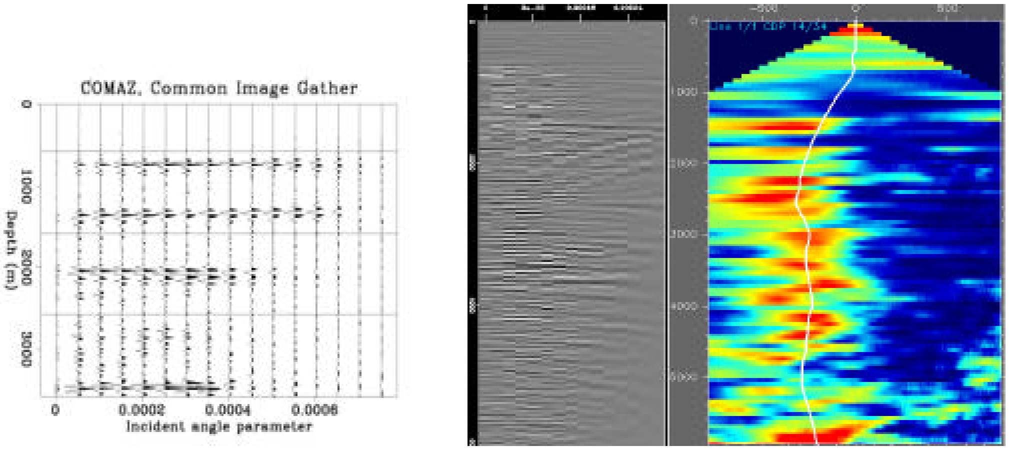

Figure 3. Left: Common Image Gather in Angle Domain. Right: Common image gather showing residual curvatures and the corresponding semblance gather with the result of automatic picking.

Common image gathers in the angle domain

Given a set of seismic data and an initial velocity model, common image gathers are generated for the volume of interest. The left panel of Figure 3 shows an Angle-domain Common Image Gather (ACIG) generated by common azimuth prestack depth migration. In this figure, the vertical axis denotes the depth while the horizontal axis denotes an incident angle parameter *p*. The incident angle parameter, also termed the ray parameter, is indicative of the angle of incidence of plane waves at each subsurface image location. Each trace in the figure corresponds to a plane wave mode.

From these ACIGs, we extract residual velocity information by scanning over angle or ray parameter. The scanning formula takes into account the relationships among migration depth, migration velocity, residual velocity, and ray parameter/angle. These scans are typically displayed as semblance panels with peaks that correspond to residual RMS values that flatten the events in the ACIGs. Figure 3, right is an MVA analysis display from a 3-D marine data set in the Gulf of Mexico. The 3-D survey covers an area 21km long and 15km wide. The initial depth interval velocity is derived from the stacking velocity. Common azimuth (wave-equation) prestack depth migration was applied to the data to generate CIGs (Figure 3 left) in the angle domain with 32 ray parameters. In the shallow part, most of the reflection events are aligned horizontally in the ACIGs. Therefore, peak semblance values are more or less aligned with zero residual in the semblance panel. Towards the bottom of the section, peak semblance values become less concentrated and reveal appreciable residuals. RMS residual velocities picked from semblance panels like this are used in vertical, normal-ray, or tomographic update depending on the level of complexity in the structures.

Once the semblance panels are generated, several methods can be used to backproject the residuals to the overburden medium and update the initial velocity model. The semblance peaks can be used directly and automatically to update the velocity, or horizons can be picked and semblances tied to these horizons. The residual semblances can then be used to update the velocity model along vertical rays, along normal rays, or tomographically.

The simplest method is **vertical updating**. The process can be automated, so that it does not require user intervention or picking of geologic horizons. Given a semblance spectrum of residual velocity, the automatic semblance picking method perturbs the initial velocity profile to search for a new velocity profile that typically follows peak semblance values at every depth. The output from this method is an updated interval velocity profile. Used alone, the method is best suited for preliminary velocity analysis in areas with slightly dipping layers. The advantage of the **automated vertical update** lies in its efficiency of processing large volume of ACIGs without human intervention. The main drawback is limited noise discrimination and limited geological constraint. A more stable and accurate approach is to tie residuals to geologic horizons and perform horizon-based updates. Geologic horizons usually exhibit strong reflectivity and good lateral continuity and can be picked from stacked image volumes.

Combining vertical update and horizon constraints results in **a horizon-based vertical update**. In this method, the Dix inversion formula (Dix, 1955) is applied to RMS residual velocity for estimation of interval residual velocity. This algorithm is computationally efficient and involves lateral smoothing of horizon-based residual velocities for each horizon. The update velocity value is the sum of the background and the calculated residual velocity at the grid point of interest. One major limitation of vertical update lies in its simplification of error back-projection methods. In the presence of steep dips, vertical back-projection can wrongfully misplace residual velocity values. With **horizon-based normal-ray updating**, perturbations occurring along a normal ray to the reflector are taken into account. A specular ray normal to the dipping reflection surface is used to approximate the wave propagation path. The improved accuracy of normal-ray backprojection comes with a slight increase in computation workload for ray tracing.

An even more accurate and computationally intensive method is **horizon-based tomographic updating**. In this method, a fan of rays with correct wave propagation trajectories is used to backproject residual velocities to the locations where the errors originated. Each tube of rays from an analysis ACIG point illuminates part of the overburden, and several overlapping ray tubes can be used to reconstruct the overburden velocity distribution in a tomographic manner. The method consists of two basic components: forward modeling and tomographic reconstruction. In the forward modeling, ray paths are determined through ray tracing from every analysis ACIG point and residual moveout as depth deviation in ACIGs is converted to residual traveltime. The influence of velocity errors on imaged reflector geometry is also taken into account. In the tomographic reconstruction part, ray paths, computed residual traveltimes, and the unknown residual slowness field comprise a linear optimization system, which is solved by the method of conjugate gradients.

Conclusion

We discuss the advantages of 3-D wave-equation migration in common azimuth domain over Kirchhoff and shot profile migration and provide synthetic and real data examples. We present several methods for using the Common Image gathers in angle domain generated by common azimuth wave-equation migration to update the initial velocity model. The migration velocity analysis tools incorporate the manual or automatic residual semblance picks into the initial interval velocity model by several choices of update: simple vertical update, vertical update constrained by horizons, normal-ray update constrained by horizons and tomographic update. The advantages of common azimuth wave-equation migration, the MVA capabilities, and the imaging improvements over Kirchhoff at the same cost, have compelled advanced technology companies to automatically include this processing step as a final imaging module in the marine data processing flow.

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