Which depth imaging method should we use? A roadmap in the maze of 3-D depth imaging

Dimitri Bevc, 3DGeo Development Inc. and Biondo Biondi, Stanford University.*

Summary

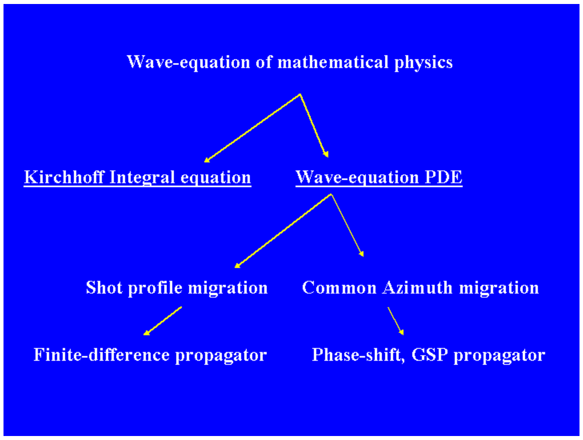
Today's explorationist is confronted with a large array of three dimensional depth imaging options, ranging from a variety of Kirchhoff implementations to a variety of wave-equation implementations. Historically, the choice of a depth migration algorithm was simple: Kirchhoff was the only practical choice. This is no longer the case. Advances in computing and clever algorithms have made wave-equation migration an economically feasible alternative. With so many choices, making the right choice of imaging method for a given objective can be a daunting task. We briefly examine the origins of the various imaging methods, describe their relative approximations, and assess their relative merits and applicability.

Introduction

The proliferation of commodity priced high-speed computers (such as Linux Clusters) and the advent of new migration formulations such as common-azimuth migration (Biondi and Palacharla, 1996) have ushered in a new interest in fully recursive wavefield downward continuation formulations, and how their results compare to nonrecursive integral results. Common nomenclature has classified these two categories of migration into: (1) Kirchhoff methods, and (2) wave-equation methods. However, this distinction is not completely accurate, because the Kirchhoff methods are in fact based on the wave equation (Figure 1). We examine the differences between the two categories of migration by looking at their mathematical formulations and examining their imaging results. We also look at various different implementations of wave-equation migration, including shot profile, plane wave, and common azimuth methods. In looking at all these approaches to solving the imaging challenge, we examine the strengths and advantages of the methods by considering the approximations that go into them, the resulting images, and the relative costs of the methods.

Kirchhoff versus wave equation methods

Three-dimensional prestack imaging has been dominated by Kirchhoff integral equation methods because up until recently, they have been the only practical methods; however, they do have their shortcomings, and a great deal of effort has been put forth to rebuild Kirchhoff and to essentially put back some of the approximations that have been made in the transformation from the full wave equation. The greatest effort in this area has been to calculate energetic (Audebert et al., 1997; Bevc, 1997; Nichols, 1996) and multivalued traveltimes. Another Kirchhoff-like class migration technique is the Gaussian beam method (Hill, 2001).



Wave-equation of mathematical physics

Kirchhoff Integral equation

Wave-equation PDE

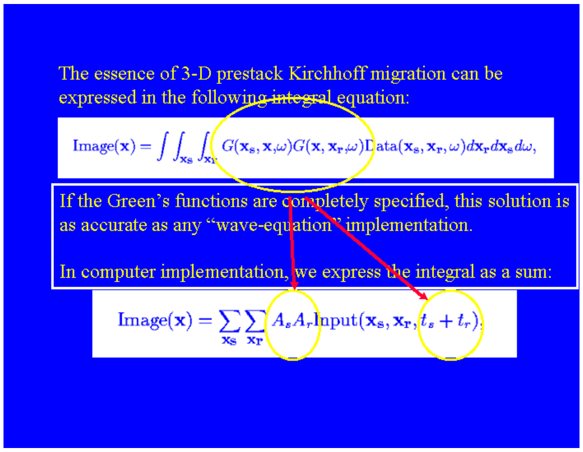
Shot profile migration

Common Azimuth migration

Finite-difference propagator

Phase-shift, GSP propagator

Figure 1: Kirchhoff and wave-equation methods are two ways to solve the wave equation of mathematical physics. The wave-equation is further subdivided into shot profile and common azimuth methods which commonly use finite difference, phase shift, or GSP propagators.



The essence of 3-D prestack Kirchhoff migration can be expressed in the following integral equation:

$$\text{Image}(\mathbf{x}) = \int\int_{\mathbf{x_s}}\int_{\mathbf{x_r}} G(\mathbf{x_s},\mathbf{x},\omega)G(\mathbf{x},\mathbf{x_r},\omega)\text{Data}(\mathbf{x_s},\mathbf{x_r},\omega)d\mathbf{x_r}d\mathbf{x_s}d\omega,$$

If the Green's functions are completely specified, this solution is as accurate as any "wave-equation" implementation.

In computer implementation, we express the integral as a sum:

$$\text{Image}(\mathbf{x}) = \sum_{\mathbf{x_s}}\sum_{\mathbf{x_r}} A_sA_r\text{Input}(\mathbf{x_s},\mathbf{x_r},t_s+t_r),$$

Figure 2: The key ellement of the Kirchhoff method is to accurately represent the Green's functions in a computer implementation as travel times and summation weighting terms.

Both Kirchhoff methods (Schneider, 1978) and recursive methods (Claerbout, 1971; Stolt 1978) came into existence at about the same time, with Kirchhoff gaining popularity for prestack applications. Kirchhoff is easy to understand, relies on a series of simple computations, and is very flexible in terms of accommodating extreme velocity variations, prestack and poststack data geometries, steep

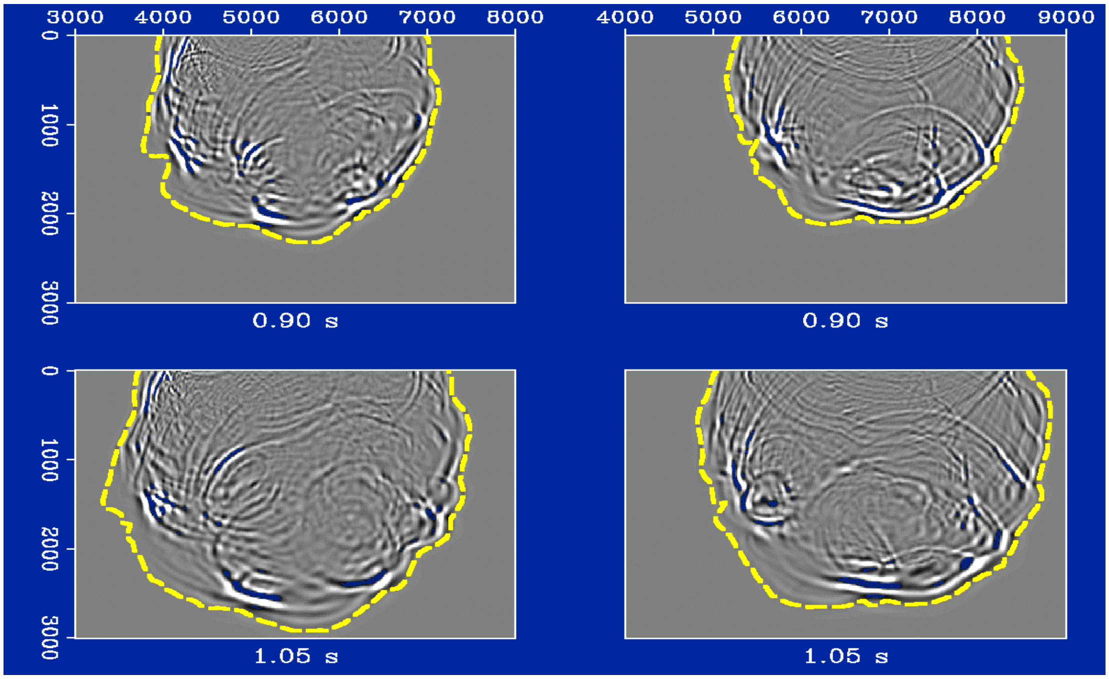


Figure 3: Green's functions for Kirchhoff migration. The wavefields are calculated at two separate surface locations and at two different times. The wavefield represents the full Green's function of the integral equation in Figure 2, and the traveltime contours (shown as yellow dashed lines) represent the travel time tables commonly used to assyptotically parameterize the Green's function. Clearly, this is adequate for the energetic first arrivals but not for more complicated propagation modes. The comlexity of the full wavefield illustrates that it would be difficult to parameterize, even with multi-branced contours. Wave-equation migration implicitly uses the entire wavefield.

dips, and most of all, 3-D data. Velocity variations and steep-dips, and prestack capabilities have all been incorporated in both phase-shift (Gazdag, 1978; Gazdag and Sguazero, 1984; Stoffa et al, 1990), and finite difference implementations of recursive methods (Claerbout, 1971). Later extensions of both finite-difference and phase shift continuation accommodate overturned rays (Claerbout, 1985; Hale, 1992), velocity variations (Hale et al, 1991), and prestack data (Popovici, 1996). In general, the recursive methods have produced higher-fidelity results when velocity variations and geological structures are complex. In the past, these fully recursive results could only be attained with much greater computer run time, making them prohibitively expensive for routine 2-D application, and out of the question for 3-D applications.

The perceived shortcoming of Kirchhoff can be illustrated in Figures 2 and 3. A Kirchhoff integral solution (Figure 2) to the wave equation hinges on the validity of the asymptotic Green's function approximation. This approximation amounts to representing the Green's functions as traveltime tables and summation weights in the computer implementation of Kirchhoff migration. The validity of this approximation is illustrated in Figure 3, which displays representative Green's functions overlaid by the time contours that represent the traveltime tables used in Kirchhoff migration. Clearly, in this example, the time contours do not accurately parameterize the entire wavefield. The motivation behind wave-equation methods is that they more directly implement the wave-equation of physics (hence their name), and therefore implicitly include all the energetic portions of the Green's functions illustrated in Figure 3.

Wave-equation migration methods

The resurgence in popularity of wave-equation methods in 3-D has been spurred by two factors: (1) Clever algorithms, and (2) fast and cheap computers. Two of the first economically feasible implementations of wave-equation migration were common azimuth migration (Biondi and Palacharla, 1996; Popovici, 2000) and offset plane-wave migration (Mosher et al., 1997). Biondi's implementation takes notice of the fact that most marine data is acquired in streamer geometry that is very nearly zero azimuth, or can be easily corrected to zero azimuth using an azimuth moveout operator (Biondi et al., 1998). This results in a 4-

D downward continuation that is extremely efficient, and is 60 times faster than the equivalent 5-D downward continuation that does not take into account the streamer geometry and the common azimuth approximation. For areas where the common azimuth approximation may be in question, this same approach can be used in a narrow or wide azimuth formulation by including some crossline offset wavenumbers in the downward continuation. The downward continuation propagator applied in common azimuth and plane wave migration is commonly some form of an extended split-step method or generalized screen propagator (Ruhl and Ristow, 1995; Le Rousseau and de Hoop, 2001; Biondi, 2001). Properly applied, these propagators are capable of imaging steep dips and in the presence of strong lateral velocity variations.

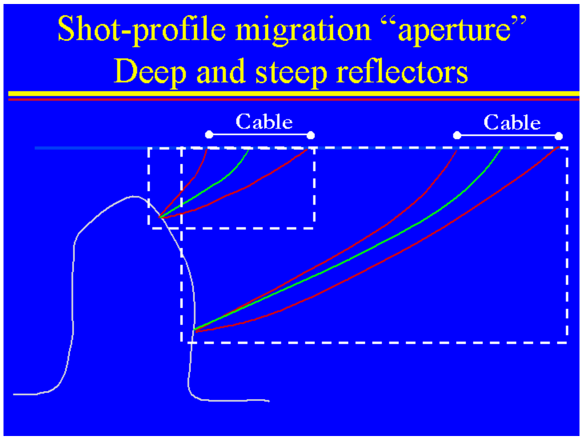

Shot-profile migration “aperture”
Deep and steep reflectors

Figure 4: Shot profile migration must retain adequate inline and crossline apperture to capture steep dips. Requirements are similar to Kirchhoff aperture. The cost of retaining the aperture, or shot profile volume is computationaly significant.

As illustrated in Figure 1, the other class of wave-equation imaging solutions is shot profile migration (Reshef and Koslof, 1986), which is commonly applied using a finite difference propagator and a cross-correlation imaging condition (Lowenthal and Hu, 1991). The shot profile approach is a full 5-D downward continuation (shot x,y, receiver x,y, and z), and therefore requires much more cpu than common azimuth migration. It’s obvious advantage is that it retains all data azimuths, so it is better suited to many land and ocean-bottom cable acquisition geometries. To get around the extreme computational cost of shot profile migration, many practitioners decimate the input data and/or reduce crossline and inline migration aperture in order to make shot profile migration economically feasible for marine streamer data. The disadvantage of decimating the shots in shot profile migration is particularly evident in the quality of prestack volumes for MVA or AVA. Even if a decimation factor of 1 to 10 produces little deterioration in the stacked image (particularly on synthetics) it creates a huge problem in the prestack image (Etgen, 2002). The danger of limiting aperture in shot profile migration is that information is lost. As illustrated in figure 4, restricting aperture in shot profile migration (or more precisely stated, the volume into which the shot record is extrapolated) can severely limit steep dip resolution.

Aside from the significant (order of magnitude) speed issue, common azimuth migration has substantial advantages in terms of amplitudes for attribute analysis (Sava et al., 2001), and the ability to generate angle gathers at no additional cost for migration velocity analysis and residual moveout (Prucha et al., 1999; Liu et al, 2001).

Imaging Examples

Figure 5 is an example of a Gulf of Mexico imaging objective. The complex salt body and underlying sediments are image using a Kirchhoff migration and common azimuth wave-equation migration. The irregular top-salt and complicated base salt offer significant imaging challenges in this area. In this case, the wave-equation migration does a superior job of imaging both top and base salt, as well as the sediments below salt. More details of this particular imaging case history are given in Fliedner et al. (2002).

We will present numerous other examples during our presentation illustrating comparisons of Kirchhoff migration and various types of wave-equation migration, including common azimuth and shot profile.

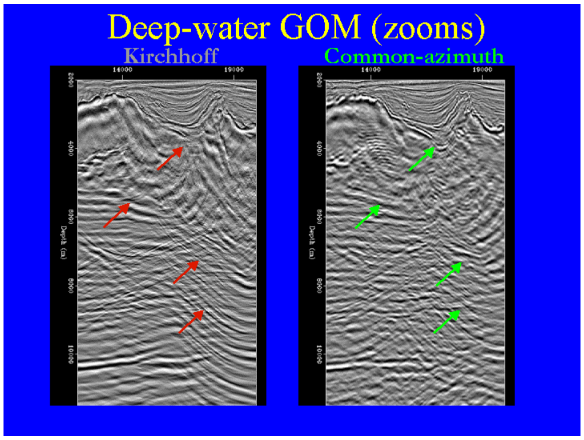

Deep-water GOM (zooms)

Figure 5: Example of Kirchhoff (left) and common azimuth wave-equation (right) migration for a Gulf of Mexico salt body. Wave-equation migration improves top-salt imaging, base of salt, and subsalt reflectors (see Fliedner et al, 2002, for more examples).

Conclusions

Based on algorithmic considerations and imaging results, we conclude that there are areas of applicability for most of the different imaging formulations. Kirchhoff and shot profile wave equation algorithms are well suited for land and ocean bottom data, while common azimuth wave equation migration is best for marine streamer data. Kirchhoff has advantages in target-oriented applications and can be used complimentary to wave equation methods to build preliminary velocity models. Nonetheless, the prospective explorationist should be aware of the strengths and weaknesses of the various imaging methods, the approximations and assumptions that are invoked, and what effect these will have on the desired outcome.

References

Audebert, F., Nichols, D., Rekdal, T., Biondi, B., Lumley, D. E. and Urdaneta, H., 1997, Imaging complex geologic structure with single-arrival Kirchhoff prestack depth migration: Geophysics, Soc. of Expl. Geophys., 62, 1533-1543.

Bevc, D., 1997, Imaging complex structures with semirecursive Kirchhoff migration: Geophysics, Soc. of Expl. Geophys., 62, 577-588.

Biondi, B., 2001, Stable wide-angle Fourier-finite difference downward extrapolation of 3-D wavefields, 71st Ann. Internat. Mtg: Soc. of Expl. Geophys., 1009-1012.

Biondi, B. and Palacharla, G., 1996, 3-D prestack migration of common-azimuth data: Geophysics, Soc. Of Expl. Geophys., 61, 1822-1832.

Biondi, B., Fomel, S. and Chemingui, N., 1998, Azimuth moveout for 3-D prestack imaging: Geophysics, Soc. of Expl. Geophys., 63, 574-588.

Claerbout, J. F., 1971, Toward a unified theory of reflector mapping: Geophysics, Soc. of Expl. Geophys., 36, 467-481.

Claerbout, J. F., 1985, Imaging the Earth's Interior, Blackwell Scientific Publications, Palo Alto.

Etgen, J, 2001, Wave Equation Technology, GSH Technical Breakfast, Geophysical Society of Houston.

Fliedner, M., Crawley, S., Bevc, D., Popovici, A.M., and Biondi, 2002, Imaging a rugose salt body in the deep Gulf of Mexico: Kirchhoff versus common azimuth wave-equation migration, 72nd, Ann. Internat. Mtg: Soc. of Expl. Geophys.,

Gazdag, J., 1978, Wave equation migration with the phase-shift method: Geophysics, Soc. of Expl. Geophys., 43, 1342-1351.

Gazdag, J. and Sguazzero, P., 1984, Migration of seismic data by phase-shift plus interpolation: Geophysics, Soc. of Expl. Geophys., 49, 124-131.

Hale, D., Hill, N. R. and Stefani, J., 1992, Imaging salt with turning seismic waves: Geophysics, Soc. of Expl. Geophys., 57, 1453-1462.

Hale, D., 1991, 3-D depth migration via McClellan transformations: Geophysics, Soc. of Expl. Geophys., 56, 1778-1785.

Hill, N. R., 2001, Prestack Gaussian-beam depth migration: Geophysics, Soc. of Expl. Geophys., 66, 1240-1250.

Le Rousseau, J. H. and de Hoop, M. V., 2001, Modeling and imaging with the scalar generalized-screen algorithms in isotropic media: Geophysics, Soc. of Expl. Geophys., 66, 1551-1568.

Liu, W., Popovici, A., Bevc, D. and Biondi, B., 2001, 3-D migration velocity analysis for common image gathers in the reflection angle domain, 71st Ann. Internat. Mtg: Soc. of Expl. Geophys., 885-888.

Loewenthal, D. and Hu, L. -Z., 1991, Two methods for computing the imaging condition for common-shot prestack migration (short note): Geophysics, Soc. of Expl. Geophys., 56, 378-381.

Popovici, A. M., 1996, Prestack migration by split-step DSR: Geophysics, Soc. of Expl. Geophys., 61, 1412-1416.

Popovici, A., 2000, 3-D wave-equation prestack depth migration, 70th Ann. Internat. Mtg: Soc. of Expl. Geophys., 493-496.

Prucha, M., Biondi, B. and Symes, W., 1999, Angle-domain common image gathers by wave-equation migration, 69th Ann. Internat. Mtg: Soc. of Expl. Geophys., 824-827.

Ruhl, T. and Ristow, D., 1995, Fourier FD migration: The missing link between phase-shift and FD migration, 65th Ann. Internat. Mtg: Soc. of Expl. Geophys., 1232-1235.

Sava, P., Biondi, B. and Fomel, S., 2001, Amplitude-preserved common image gathers by wave-equation migration, 71st Ann. Internat. Mtg: Soc. of Expl. Geophys., 296-299.

Schneider, W. A., 1978, Integral formulation for migration in two-dimensions and three-dimensions: Geophysics, Soc. of Expl. Geophys., 43, 49-76.

Stoffa, P. L., Fokkema, J. T., de Luna Freire, R. M. and Kessinger, W. P., 1990, Split-step Fourier migration: Geophysics, Soc. of Expl. Geophys., 55, 410-421.

Mosher, C. C., Foster, D. J. and Hassanzadeh, S., 1997, Common angle imaging with offset plane waves, 67th Ann. Internat. Mtg: Soc. of Expl. Geophys., 1379-1382.

Nichols, D. E., 1996, Maximum energy traveltimes calculated in the seismic frequency band: Geophysics, Soc. of Expl. Geophys., 61, 253-263.

Reshef, M. and Kosloff, D., 1986, Migration of common shot gathers: Geophysics, Soc. of Expl. Geophys., 51, 324-331.

Stolt, R. H., 1978, Migration by Fourier transform : Geophysics, Soc. of Expl. Geophys., 43, 23-48.

Acknowledgements

We thank Unocal Corporation for permission to show the Gulf of Mexico data results.