Application of 2nd generation wavelets to seismic imaging

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Summary

We study the sensitivity of seismic imaging to lossy data compression using a new generation of directional, nonseparable wavelets developed by Donoho et al [1, 3, 5, 6, 7], and compare these results to those obtained using a conventional biorthogonal wavelet compression technique. In our study we use two streams of minimally processed seismic data: the SEG-EAEG synthetic salt model and a field data set. Our objective is to show that features of the imaged data are much better preserved at compression ratios of 20:1 and higher with the newer technology. The advent of the Grid has emphasized the need for the improved compression and effective transmission of large data sets, such as seismic surveys, over the Internet. It is toward this purpose that this work is directed.

Introduction

The usual procedure in wavelet compression is to process the data through the following steps:

1. Use separable wavelets (tensor products of 1-D wavelets, and of wavelets and scaling functions) to transform the data into a set of local *detail coefficients* at various scales of resolution

2. Threshold or quantize the detail coefficients, setting to zero the coefficients of *small* amplitudes.

3. Apply standard lossless compression techniques such as runlength and entropy encoding to the thresholded quantized data.

There is an extensive literature on this type of wavelet compression for reducing the storage and transmission requirements of large multidimensional seismic datasets [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26].

In this paper we investigate a newer technology: bases of *directional wavelets*, nonseparable higher dimensional wavelet-like bases that represent more efficiently the directional features of the data [1, 3, 5, 6, 7]. To our knowledge, the first attempt to apply this technology in seismic imaging was by Hermann [8] where he used fractional spline directional wavelets to improve the signal-to-noise ratio and resolution of seismic images. Our objective is to investigate the effectiveness of directional wavelets for the compression of seismic data by assessing its effects on seismic imaging. To this purpose we compare the imaging results obtained with data compressed with an orthonormal ridgelet basis and with a conventional biorthogonal basis. Our aim is to show - contingent to nonobservable degradations in the quality of the imaging results – that much better rates of compression can be achieved with the newer technology.

A scenario thus becomes feasible where 3D seismic data is compressed and transmitted, through the Internet, between nodes in a Grid of remote computational and storage resources, and where the data is reconstituted and imaged on Grid nodes as they become available throughout the life of an imaging project. A motivation for our work is thus partly to address the increasing demand of computational and data resources for 3-D imaging applications, and the ensuing need of providing geoscientists with secure, available, flexible, and computationally efficient Grid resources.

The accurate reconstruction of 3-D images from 3-D seismic data today requires (1) the handling of huge data volumes, in the order of 10-15 terabytes for a single modern marine 3-D survey, and (2) the application of sophisticated computation-intensive 3D parallel applications such as wave equation migration. Some difficult imaging projects are beyond the reach of the computational resources available to the industry. Furthermore, new technologies will dramatically increase the size of data, require more expensive algorithms, and aggravate this situation even more.

The near future is thus likely to bring hundreds of terabytes of seismic data for medium to large 3-D surveys, volumes that are too large to process routinely today, even for the few largest data processing centers. In this environment it will become essential for geoscientists to harness Grid resources in a computationally efficient manner. The utilization of the Grid for seismic imaging, however, will require an effective mechanism for transmitting very large data sets over the Internet between a changing set of remote computational and storage resources. Our aim is to evaluate the effectiveness of wavelet compression for this objective.

Wavelet compression methods

Wavelet analysis allows the representation of a function defined on a spatial domain in terms of its coefficients with respect to a set of basis functions localized both in space and in scale or frequency (an analogy is the way a musical piece is represented by its written score, using time-localized notes of varying frequencies with varying amplitudes). The basis functions, which can be selected to achieve optimal sparse representations of target classes of functions, can be mutually orthogonal - as is the case with the Daubechies wavelets - or biorthogonal with respect to a dual set of basis functions, or can even constitute a *frame*, a possibly redundant set of functions allowing for nonunique representations. In other words, it is not necessary that the basis functions constitute an orthonormal base in the traditional sense, nor that they even be linearly independent. The properties we expect from them are the following:

- The basis functions should be well localized in space and in frequency or scale. [As is well known, simultaneous localizations in space and in frequency are subject to trade-offs implicit in Heissenberg's uncertainty principle.]

- The basis functions should allow for the series representation of broad classes of functions.

- The coefficients in the series representations should be sparse, or decay quickly, for functions of specified characteristics.

- The computation of coefficients should have a low order of computational complexity.

While 1-D wavelets applied to 1-D data are satisfactory in these respects, the conventional approach of using tensorial bases built from 1-D bases in higher dimensions is not. The tensorial bases exhibit directional biases along directions defined by the edges and diagonals of the data cube, and are consequently unsuited for data that exhibit complex directional behavior, such as seismic data where discontinuities usually present themselves along piecewise smooth curves.

Compression Processing

We assess the effect of two compression methods applied to the data:

A method based on separable basis functions, built from.1-D compactly supported biorthogonal spline wavelets. As is well known biorthogonal wavelets have symmetry and allow for exact reconstructions, but have different sets of mirror filters for decomposition and freconstruction [4, pp. 271-280]. Furthermore, they allow for the sparse representation of piecewise polynomial functions of up to a maximum degree dependent on the length of the filters. The most effective systems evaluated by us had filters of moderate lengths (up to length 16). By applying global thresholding to the transformed data, we reduced the number of nonzero elements in the transformed data to a small percentage of the original number. We then reconstituted and imaged the data. Both original and transformed data elements were represented as 32 bit floating point numbers. The computational complexity of the procedure was O(N) for data of size N x N.

A method based on an orthonormal ridgelet basis [5, 6], i.e. a basis of nonseparable directional wavelets obtained by using fractional derivatives of 1-D Meyer wavelets [4, pp. 114-119] as *ridge profiles,* and by averaging the corresponding *ridge functions,* at different localities and in different scales, along a range of directions with angular weights given by Battle-Lemarie scaling functions [4, pp. 146-152]. These functions form an orthonormal basis in $L^2(R^2)$, allow for optimally sparse representations of data exhibiting singularities along curved directions, and are consequently well adapted to the characteristics of seismic data. As in the biorthogonal case, we apply global thresholding to the transformed data, setting to zero the elements of large amplitudes and thus reducing the number of nonzero elements in the transformed data to a small percentage of the original number of data elements. As before, the thresholded transformed data is reconstituted and imaged, and the imaged results compared to those obtained from the original data. The dominant computational task are FFTs; the computational order of complexity is O(N log N).

Migration Imaging Examples

We tested the effect of compression by running two data sets through the biorthogonal spline wavelet process as described in the previous section. The data were then imaged using Kirchhoff migration. In both cases, the pre and post compression data were run through the exact same processing flow, using the same migration parameters, and the some velocity for pre and post compression.

The first example is a 2-D land data set with relatively mild structure. In this example the pre compression image (Figure 1) and post compression image (Figure 2) are difficult to tell apart. Differencing the two image files (middle panel of Figure 3) reveals a largely uncorrelated distribution with significantly less energy than the images.

The second example is a 2-D synthetic generated by taking a slice out of the SEG/EAGE 3-D model. In this case, the pre compression image (Figure 4) and post compression image (Figure 5) do show significant differences, especially in the steep dip imaging. Figure 6 shows the difference panel along with the pre and post compression migration result. The image differences arise because of the presence of steep dips that do not comply with the directionality of the biorthogonal spline wavelets, and because of the sparcity of the synthetic data. In contrast, the results, we will show based on a ridgelet basis handle the steep dips much better and allow much higher compression ratios.

Conclusions

Nonlinear approximations of seismic data obtained by thresholding the ridgelet coefficients result in rapidly converging series in the presence of singularities along curved directions as is the case with seismic data, particularly in the SEG/EAGE salt model. The convergence is at the maximum possible rate, which in our view makes the ridgelet-based method near optimal for compressing seismic data. The results we will present validate this view by overcoming directional shortcomings of separable wavelets and by thus allowing for much higher effective compression ratios of seismic data.

Figure 1. Migration of uncompressed data.

Figure 2. Migration of 90% compressed data.

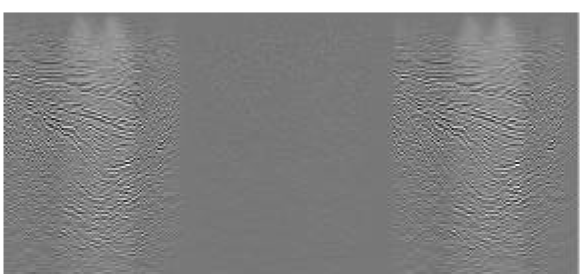

Figure 3. Left panel is migration of uncompressed data (Figure 1) , center panel is difference between migration of uncompressed data and 90% compressed data, and third panel is migration of 90% compressed data (Figure 2). The difference is lower amplitude than the images and relatively uncorrelated.

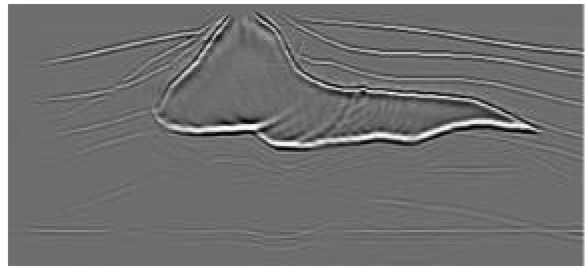

Figure 4. Migration of uncompressed data.

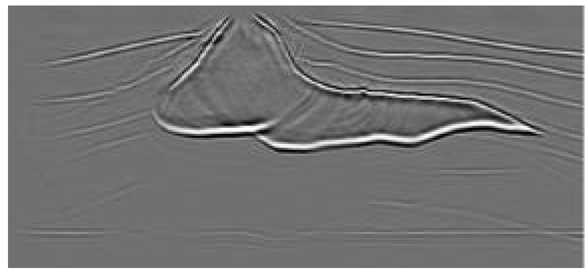

Figure 5. Migration of 95% compressed data.

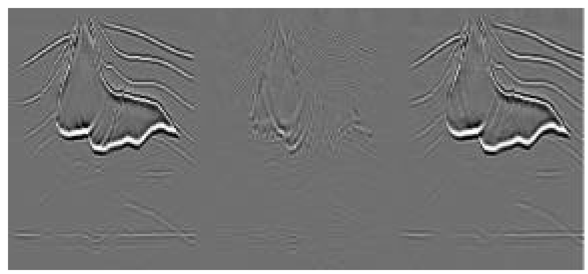

Figure 6. Left panel is migration of uncompressed data (Figure 4) , center panel is difference between migration of uncompressed data and 95% compressed data, and third panel is migration of 95% compressed data (Figure 5). The difference is lower amplitude than either image, but the steep dip targets clearly create greater differences in the pre and post compression images than observed in Figures 1 through 3.

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