Compression of seismic data using ridgelets

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Summary

Ridgelets are wavelet-like bases developed by Donoho *et al*. ([1], [2], [3]) for the sparse representation of data with directional feature. The purpose of this study is to assess the effectiveness of ridgelet bases for the compression of seismic data. This work is a continuation of [4] where preliminary results using 2-D ridgelets to compress prestacked 3-D data were presented. Multidimensional ridgelets are standard multidimensional wavelets defined in a multidimensional pseudo-Radon domain. The 3D ridgelet transform operates on an entire 3-D volume, instead of on a set of 2-D slices of the data as it's 2-D counterpart, thus allowing for data coherency in three dimensions to be exploited. Our results suggest that compression ratios much higher than what can be attained with a 2-D procedure can be attained using 3-D ridgelets. In this pap er we show results of ridgelet compression on a shallow marine prestacked dataset, and give an outline of a compression system based on ridgelets in 3-D.

Introduction

Wavelets are an ubiquitous component of any state-of-the-art image compression system. The success of wavelets is due to the following:

- Wavelets are functions localized both in the physical and frequency domains (but contingent to the Heissenberg Uncertainty Principle).
- The wavelet coefficients of smooth functions decay steeply with increasing resolutions. This allows for their sparse approximation and efficient compression.
- Wavelets give an efficient representation of point singularities in the data.
- The wavelet transform can be implemented using a fast algorithm.
- Wavelet bases can be chosen to match certain characteristics of the target data. This allows for adaptive compression strategies using wavelets.

A set of 2-D biorthogonal wavelet basis functions using the optimized filters of Donoho and Ergas [5] are shown in Figure 1.

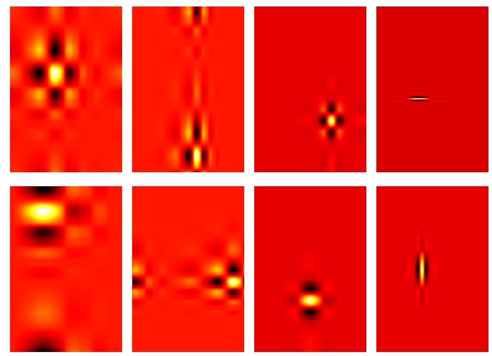

Figure 1. Tensorial biorthogonal wavelets in 2-D

Wavelets in higher dimensions, as those shown in Figure 1, are usually the tensor products of one-dimensional wavelets. The disadvantages of tensorial wavelets is that they exhibit a directional bias in the directions of the coordinate axes, and that because of this, provide inefficient representions of line singularities.

These shortcomings are overcome with directional wavelet systems such as the ridgelet basis functions shown in Figure 2.. Ridgelets provide sparse coding of images with edges. In addition to being localized in the physical and frequency domains, they are aligned along a wide range of directions, and thus give compact representions of line singularities.

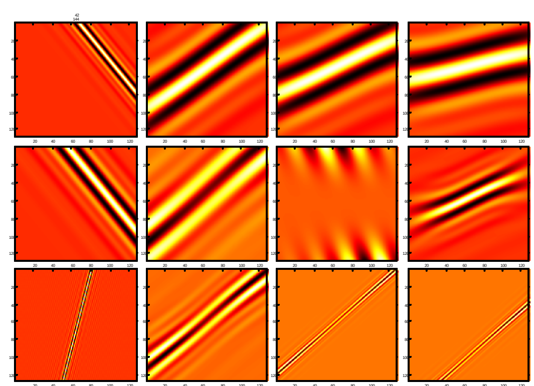

Figure 2 . A set of 2-D ridgelet basis functions at different scales, localities, and with different preferential directions.

Compression using higher dimensional ridgelets

Data compression is an essential component of any system for processing and imaging seismic data on a dedicated network or on the Internet. A system for 3-D seismic imaging on the Grid, by providing users a collaborative environment and a pool of interconnected data, computational, and human resources, promises major increases in efficiency and capability over current practices. The motivation for our work is the necessity of extremely high compression ratios to enable seismic imaging on the Internet. In [4] we investigated the effectiveness of 2_D ridgelets to compress prestacked seismic data and obtained satisfactory results at compression ratios of 50:1. while these results were encouraging, a Grid-based seismic imaging system requires much higher compression ratios. It is toward this goal that this work is directed.

Procedure

The theoretical development of the ridgelet transform in 2-D is outlined in [1,2,3].

Our 2-D ridgelet transform of a 3-D dataset proceeds through the steps given below:

Step 1. the data is divided into 2-D slices which are further subdivided into a set of overlapping segments which are processed separately. These segments are obtained using a partition of unity of the data to ameliorate edge effects.

Step 2. A 2-D FFT is applied to each pannel.

Step 3. The Fourier-transform of the pannel data is interpolated in the frequency domain using a trigonometric polynomial, and then specified on a 2-D pseudopolar grid as shown in Figure 3..

Step 4. The Projection-Slice Theorem [6] is applied in the frequency domain to obtain a 2-D pseudo-Radon or Slant-Stacked Transform of the data,

Step 5. Finally a standard multidimensional wavelet transform (*e.g*. Meyer wavelet transform [7]) is applied in the Radon domain.

The inverse ridgelet transform follows the steps in a reverse order, and recreates the data exactly. The 3-D ridgelet transform proceeds through the same steps except that these are all applied in 3-D instead of 2-D.

A compression system based on the ridgelet transform includes the steps listed above, plus a thresholding/quantization step and a lossless compression step (entropy or Huffman-type encoding) applied to the set of ridgelet coefficients. The inverse ridgelet transform when applied to the compressed data inverts the process and delivers an approximate reconstruction of the uncompressed data.

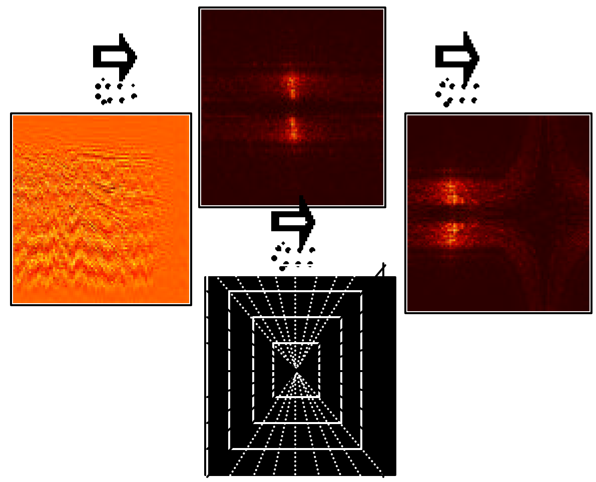

Figure 3. A 2-D panel of input data is shown on the left. It's 2-D amplitude spectrum is shown on top. The pseudopolar grid system is shown below. The specification of the amplitude spectrum to the pseudopolar grid is shown on the right.

The pseudo-Radon transform is implemented using a Discrete Projection Slice Theorem relating it to the pseudopolar Fourier transform: The 1-D Fourier transform of the columns in the Radon domainis equal to the 2-D Fourier transform of the image evaluated on the line that the projections were taken.

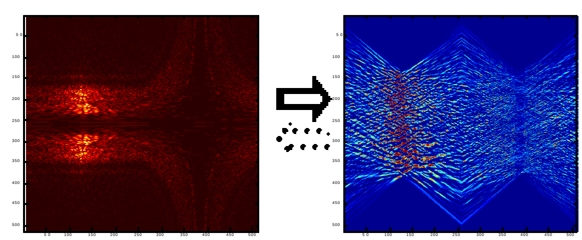

Figure 4. The pseudo-Radon transform via the Projection Slice Theorem.

The two halves of the panel on the right in Figure 4 represent the projections of the data on "horizontal lines", or lines in the pseudopolar grid of slopes between –1 and 1. The panel on the right represents the projections on "vertical lines" of slopes greater than 1 and less than –1.

Compression using higher dimensional ridgelets

The final step is the application of a conventional wavelet transform, in our case Meyer wavelets, on the Radon domain.

The dominant computational task in the ridgelet transform are the FFTs needed in the calculation of the pseudo-Radon and Meyer wavelet transforms. An important contributor is the trigonometric interpolation of the 3-D Fourier transform to obtain values on nodes in the pseudopolar grid. The order of computational complexity for an n-dimensional ridgelet transform on a data cube of size N^n is $O(N^n \log N)$,.

Results

The data used in our example was shallow marine data: a standard 2-D seismic line to which first arrival mute was applied. The time sampling was 4ms, offset sampling was 100m, minimum offset was 25m, and Cdp spacing was 25 m. The success of the compression procedure was established by comparing the migrated uncompressed and compressed data, and assessing the loss of coherent information in the migrated compressed data. The standard signal-to-noise ratio of the compressed unmigrated data was not a good indicator since an important effect of ridgelet as well as wavelet compression is the suppression of noise. The processing was Kirchoff migration with traveltime calculations obtained using a fast marching method. The effects of 2-D compression on unmigrated and migrated data are shown in Figures 5 and 6.

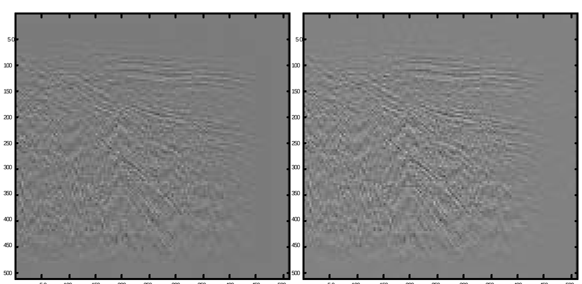

Figure 5 Ridgelet compression on a segment of unmigrated data. The panel on the left is the uncompressed data. The one on the right is the reconstituted.compressed data.The compression ratio is 50:1. The signal-to-noise ratio is 8.

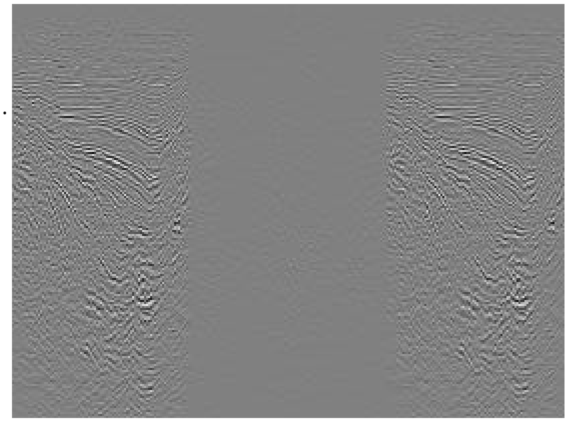

Figure 6. Effect of ridgelet compression on migrated data. The panel on the left shows the migrated uncompressed data, the panel on the right the migrated compressed data. The compression ratio was 50:1. The signal-to-noise ratio was 13. The panel in the center shows the difference as consisting mostly of random noise.

Conclusions

A critical component of any Internet-based seismic imaging is the capability to faithfully represent data at extremely high compression ratios. Although compression ratios of 40-to-1 are feasible with existing wavelet compression technologies, much higher compression ratios are needed to transfer large seismic surveys through the Internet or even a high-performance dedicated network. The advantage of 3-D ridgelets is that they pick spatio/temporal details of the data, over a range of scales and directions in threee dimensions. As such, 3-D ridgelets are a better analysis tool for layered data, such as seismic data, and a more effective tool for their compression. Based on preliminary results we believet that compression ratios of the order of 100-to-1 are possible using ridgelets in 3-D or 4-D.

Our results indicate the following:

- Compression ratios substantially higher than what is possible using standard methods are possible using multidimensional ridgelets.
- A lossless compression procedure specific to ridgelet-encoded data is needed to further improve efficiency.
- Depth imaging on the Grid appears feasible in a commercial scale using advanced wavelet compression in 3-D and 4-D.

References

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