Which depth imaging method should you use? A road map through the maze of possibilities

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Today's explorationist is confronted with a large array of three-dimensional depth imaging options, ranging from a variety of Kirchhoff implementations to a variety of wave-equation implementations. Historically, the choice of a depth migration algorithm was simple: Kirchhoff was the only practical option. This has changed. Advances in computing and clever algorithms have made wave-equation migration an economically feasible alternative. However, so many choices mean that making the right choice of imaging method for a given objective can be a daunting task. In this article we briefly examine the origins of the various imaging methods, describe their relative approximations, and assess their relative merits and applicability.

The proliferation of commodity priced high-speed computers (such as Linux clusters) and the advent of new migration formulations such as common-azimuth and offset plane wave migration have generated new interest in fully recursive wavefield downward continuation formulations, and how their results compare to nonrecursive integral results. Common nomenclature has classified these two categories of migration into: (1) Kirchhoff methods, and (2) wave-equation methods. However, this distinction is not completely accurate, because the Kirchhoff methods are in fact based on the wave equation (Figure 1). We examine the differences between the two categories of migration by looking at their mathematical formulations and examining their imaging results. We also look at various different implementations of wave-equation migration, including shot profile, plane wave, and common azimuth methods. In looking at all these approaches to solving the imaging challenge, we examine the strengths and disadvantages of the methods by considering the approximations that go into them, the resulting images, and the relative costs.

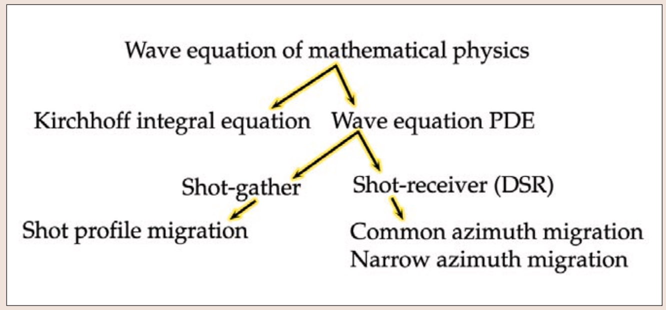

Figure 1. Kirchhoff and wave-equation methods are two ways to solve the wave equation of mathematical physics. Wave equation migration is further subdivided into shot profile and shot receiver (also known as double square root—DSR—methods). These approaches commonly use finite difference, phase shift, or generalized screen propagators.

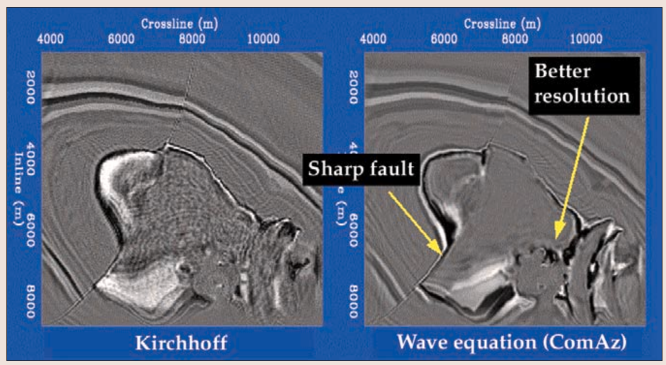

Figure 2. 3D depth slice through the SEG/EAGE model illustrating the advantage of wave-equation migration over Kirchhoff.

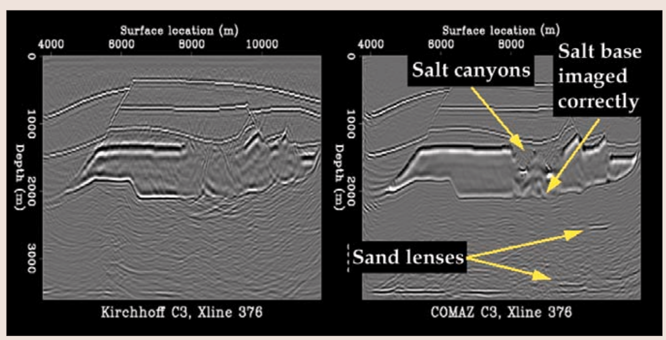

Figure 3. 3D section through the SEG/EAGE model illustrating the advantage of wave-equation migration over Kirchhoff.

Kirchhoff versus wave-equation methods. Three-dimensional prestack imaging has been dominated by Kirchhoff integral equation methods because, until recently, Kirchhoff has been the only practical method. Kirchhoff methods, however, have shortcomings, and a great deal of effort has been made to overcome some of the approximations that have been made in the transformation from the full wave equation. The greatest effort in this area has been to calculate energetic and multivalued traveltimes. Along these lines, another Kirchhoff-like migration technique is the Gaussian beam method.

Both Kirchhoff methods and recursive methods came into existence at about the same time (the late 1970s), with Kirchhoff gaining popularity for prestack applications. Kirchhoff is easy to understand, relies on a series of simple computations, and is very flexible in terms of accommodating extreme velocity variations, prestack and poststack data geometries, steep dips, and, most of all, 3D data. Velocity variations and steep dips, and prestack capabilities have all been incorporated in both phase-shift and finite-difference implementations of recursive methods. Later extensions of both finite-difference and phase shift continuation accommodate overturned rays, velocity variations, and prestack data. In general, the recursive methods have produced higher-fidelity results when velocity variations and geologic structures are complex. In the past, these fully recursive results could only be attained with much greater computer run time, making them prohibitively expensive for routine 2D application, and out of the question for 3D applications. Examples of Kirchhoff and wave equation results on an industry-standard benchmark data set are shown in Figures 2 and 3.

The perceived shortcoming of Kirchhoff are illustrated in Figures 4 and 5. A Kirchhoff integral solution (Figure 4) to the wave equation hinges on the validity of the asymptotic Green's function approximation. This approximation amounts to representing the Green's functions as traveltime tables and summation weights in the computer implementation of Kirchhoff migration. The validity of this approximation is illustrated in Figure 5, which displays representative Green's functions overlaid by the time contours that represent the traveltime tables used in Kirchhoff migration. Clearly, in this example, the time contours do not accurately parameterize the entire wavefield.

The motivation behind wave-equation methods is that they more directly implement the wave-equation of physics (hence their name), and therefore implicitly include all the energetic portions of the Green's functions illustrated in Figure 5.

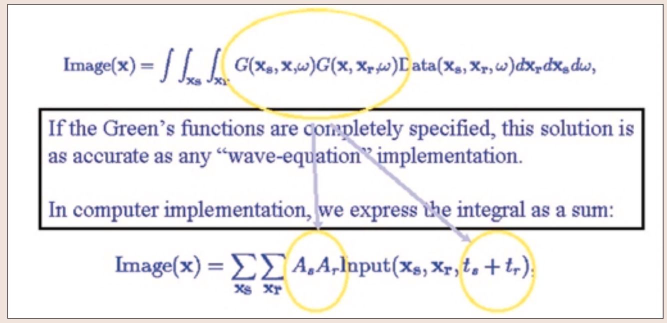

Figure 4. The key element of the Kirchhoff method is to accurately represent the Green's functions in a computer implementation as traveltimes and summation weighting terms.

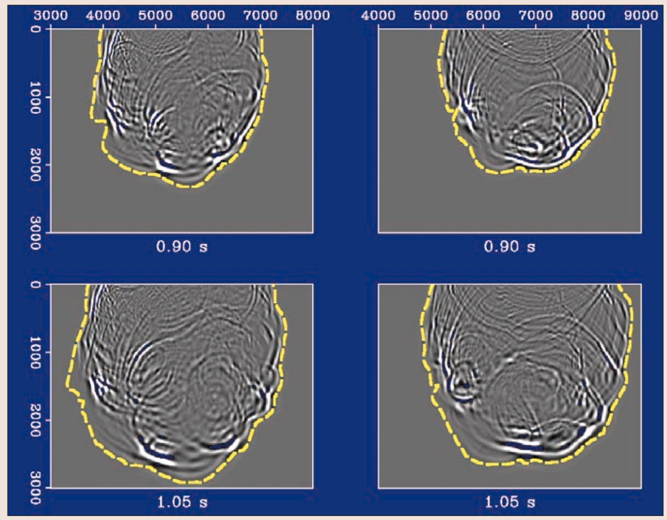

Figure 5. Green's functions for Kirchhoff migration. The wavefields are calculated at two separate surface locations and at two different times. The wavefield represents the full Green's function of the integral equation in Figure 4, and the traveltime contours (shown as yellow dashed lines) represent the traveltime tables commonly used to assymptotically calculate the Green's function. Clearly, this is adequate for the energetic first arrivals but not for more complicated propagation modes. The complexity of the full wavefield illustrates that it would be difficult to parameterize, even with multibranched contours. Wave-equation migration implicitly uses the entire wavefield.

Resurgence of wave-equation migration methods. The resurgence in popularity of wave-equation methods in 3D has been spurred by two factors: (1) clever algorithms, and (2) fast and cheap computers. Wave-equation methods can be generally grouped by their computational domain (shot-profile, source-receiver, survey sinking, plane wave) and by the numerical method used to extrapolate or downward continue the wavefield—finite difference, frequency domain, generalized screen propagator (GSP), Fourier finite difference (FFD), etc. In addition, wave-equation methods can be solutions of the two-way wave equation (reverse time) or the one-way wave equation. Two-way wave equation methods are computationally more expensive, although they do promise potential advantages for imaging overturned rays. In this discussion, we limit ourselves to more commonly available solutions based on one-way wave equation downward continuation, and we look at the differences in wave-equation methods based on the characterization of their computational domain. In terms of classification by numerical extrapolation, we simply assert that any choice of migration method must incorporate an extrapolator capable of handling strong lateral velocity variations and steep dips. Most commercial applications should incorporate these essential elements, and the technical literature is full of detailed analysis of the various methods.

Shot profile compared to source-receiver downward continuation. One of the most common computational domain divisions between wave-equation methods in the industry today is that between shot-profile migration (SPM) and source-receiver migration; source-receiver is also commonly referred to as survey sinking method (SSM) or double-square root (DSR) method, and despite the name, is commonly applied in the midpoint-offset domain.

To understand the two methods, we briefly outline how they work. With SPM each shot record is migrated individually into an image volume by: (1) downward continuing the receiver wavefield, (2) downward continuing the source (i.e., modeling the shot), and (3) imaging by cross-correlation of the two wavefields and extracting the zero lag.

Source-receiver downward continuation is performed by applying the DSR equation at each depth step to simultaneously: (1) downward continue the receiver wavefield, and (2) downward continue the source wavefield, and (3) applying the imaging condition at each depth step by extracting the wavefield at zero time and zero offset.

The observant reader will note that steps one and two are similar. In fact, the downward continuation of receiver and source wavefield commutes, and the order can be rearranged. With some manipulation of equations, it can be shown that the two methods are mathematically equivalent. Therefore, shot-profile and source-receiver downward continuation are theoretically equivalent. This means, that properly implemented, the two methods should yield equivalent accuracy and comparable imaging results. The difference then becomes purely an engineering issue and, as we describe below, for marine data, source-receiver methods offer significant opportunities for algorithmic efficiency based purely on the computational domain.

Two of the first economically feasible implementations of wave-equation migration were common azimuth migration and offset plane-wave migration. The 1998 implementation of Biondi et al. takes notice of the fact that most marine data are acquired in streamer geometry that is very nearly zero azimuth, or can be easily corrected to zero azimuth using an azimuth moveout operator. This results in a 4D downward continuation that is extremely efficient, and is orders of magnitude faster than the equivalent 5D downward continuation that does not take into account the streamer geometry and the common azimuth approximation. For areas where the common azimuth approximation may be in question, this same approach can be used in a narrow- or wide-azimuth formulation by including some crossline offset wavenumbers in the downward continuation. The downward continuation propagator applied in common azimuth and plane wave migration is commonly some form of an extended split-step method or generalized

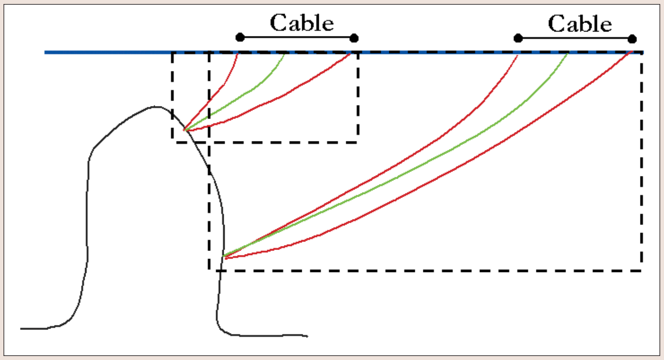

Figure 6. *Shot-profile migration must retain adequate inline and crossline aperture to capture steep dips. Requirements are similar to Kirchhoff aperture. The cost of retaining the aperture, or shot-profile volume, is computationally significant.*

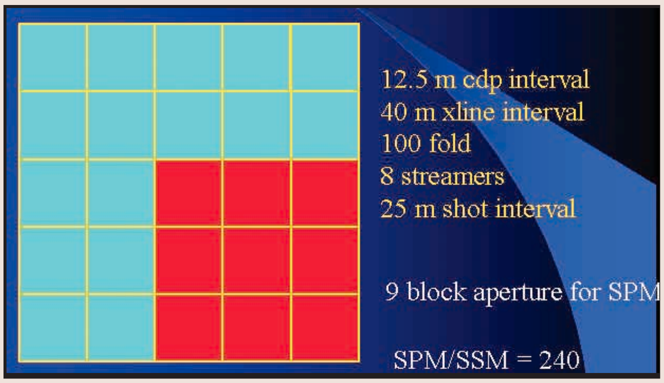

Figure 7. *Illustration of geometry for imaging nine blocks of GOM data. One block imaging halo requires input of 25 blocks. SPM/SSM cost ratio is 240, and SPM has restricted aperture.*

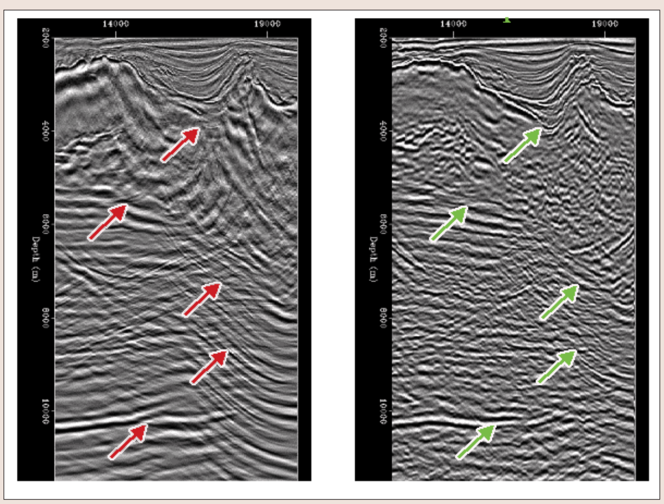

Figure 8. *Example of Kirchhoff (left) and common azimuth wave-equation (right) migration for a Gulf of Mexico salt body. Wave-equation migration improves top-salt imaging, base of salt, and subsalt reflectors (see Fliedner et al., 2002, for more examples).*

screen propagator. Properly applied, these propagators are capable of imaging steep dips and strong lateral velocity variations.

Computational cost of SPM versus SSM/DSR. As illustrated in Figure 1, the other class of wave-equation imaging solutions is SPM, which is commonly applied using a finite-difference propagator and a cross-correlation imaging condition. The shot profile approach is a full 5D downward continuation—shot (x,y), receiver (x,y), and z—and therefore requires much more computing power than common azimuth migration. Its obvious advantage is that it retains all data azimuths, so it is better suited to many land and ocean-bottom cable acquisition geometries.

To get around the extreme computational cost of shot-profile migration, many practitioners decimate the input data and/or reduce crossline and inline migration aperture to make shot-profile migration economically feasible for marine streamer data. The disadvantage of decimating the shots in shot-profile migration is particularly evident in the quality of prestack volumes for migration velocity analysis (MVA) or amplitude variation with angle (AVA). Even if a decimation factor of 1-to-10 produces little deterioration in the stacked image (particularly on synthetics), it creates a huge problem in the prestack image. The danger of limiting aperture in shot-profile migration is that information is lost. As illustrated in Figure 6, restricting aperture in shot-profile migration (or more precisely stated, the volume into which the shot record is extrapolated) can severely limit steep dip resolution.

Common azimuth (ComAz) is based on the observation that marine streamer data are collected along relatively narrow streamer arrays, and makes the assumption that multistreamer data can be represented by an equivalent (after rebinning or azimuth moveout) data set that is purely zero azimuth. The method further assumes that migrated energy does not rotate in azimuth during the downward continuation process of migration imaging. These assumptions are generally good, but an exception occurs for the case of steeply dipping imaging targets that are at an azimuth of 45° to the data acquisition geometry. Under these conditions the ComAz assumptions break down, and the resulting image is degraded (this image degradation is typically manifested as a reflector mispositioning or as an apparent velocity error).

Narrow azimuth migration (NarAz) addresses this particular issue by allowing the data to retain the narrow azimuth range with which they are acquired. Instead of assuming that the data are all zero-azimuth and are not allowed to rotate during downward continuation, NarAz assumes data are acquired over a narrow crossline azimuth range, and that the data are allowed to rotate over the given azimuth range. When NarAz is implemented to allow an adequate number of crossline azimuths (typically 3-16), it will capture all recorded propagation events and image them accurately for a computational cost that is substantially less than that of SPM.

The cost ratio of SPM to SSM can be calculated by considering the geometry of the input and output computational grids. For implementations that use the same type of propagator, this ratio is:

$$\frac{SPM}{SSM} = \frac{\text{output grid points} * \text{shot input grid points} * 2}{\text{offsets} * \text{cdp input grid points}}$$

Figure 7 illustrates the relative geometries of SPM and SSM migration for a typical Gulf of Mexico (GOM) scenario. A derivation of this ratio, and additional scenarios are available from the authors.

Aside from the significant (order of magnitude) speed issue, ComAZ and NarAz migration have substantial advantages in terms of amplitudes for attribute analysis, and the ability to generate angle gathers at no additional cost for migration velocity analysis and residual moveout.

The greater speed of processing offered by NarAz over SPM translates into shorter turnaround times. Provided the turnaround times are sufficiently short, the processing, depth imaging and perhaps the interpretation phases of a 3D seis-

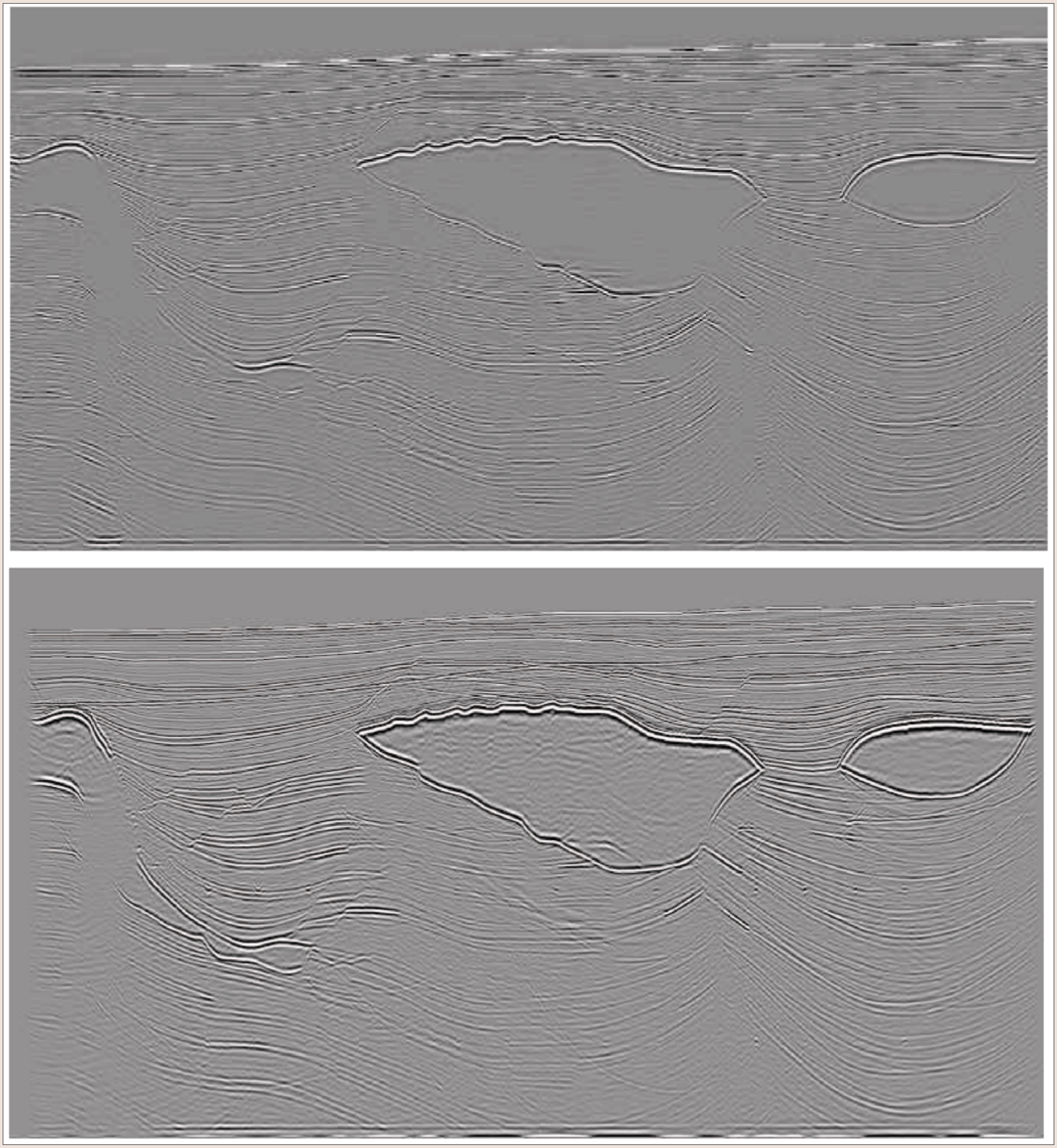

Figure 9. Migration of Pluto 1.5 synthetic data. Shot profile on the top, survey sinking on the bottom.

mic survey may allow for several, very significant iterations and consequently better results. This latter speed-of-processing advantage and access to much larger blocks of survey data may enable a significant change in imaging, target definition and characterization. This is not feasible with conventional, or older and slower algorithms (such as SPM).

Imaging examples. Figure 8 shows a GOM imaging objective. The complex salt body and underlying sediments are imaged using a Kirchhoff migration and common azimuth wave-equation migration. The irregular top salt and complicated base salt offer significant imaging challenges in this area. In this case, wave-equation migration does a superior job of imaging both top and base salt, as well as the sediments below salt. More examples and details of this particular imaging case history are given in Fliedner et al. (2002).

The Pluto 1.5 synthetic data set of the SMAART JV simulates a deepwater GOM imaging objective with steep dips and significant velocity contrast. Figure 9 shows SPM and DSR migrations of Pluto 1.5. Figure 10 is a closeup of the central salt body showing the imaging of steep dips and sediment truncations against the salt flanks. The Pluto synthetic is sampled more densely in receivers than in shots, and is therefore better suited geometrically to SPM than to SSM/DSR which operates in the midpoint-offset domain; nonetheless, both migrations produce similar results, imaging subsalt sed-

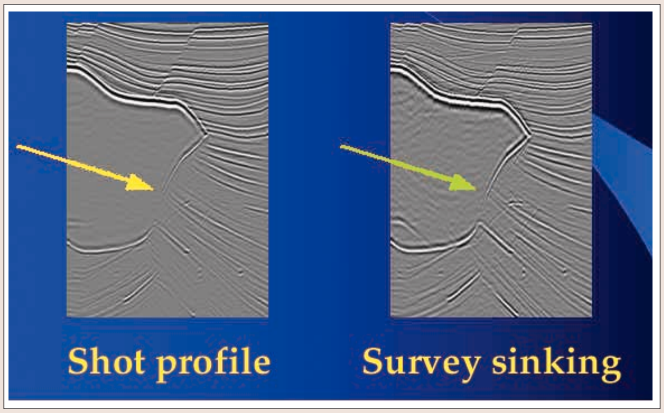

Figure 10. Closeup of Pluto 1.5 synthetic migration results comparing imaging of steep dips and continuation of sediments up to the salt flank.

	Amp	Land OBC	Marine	Recon image	Target	MVA	Image accuracy	Speed	Final image
Kirchhoff	x	+	x	+	+	x	-	x	-
Shot Profile	-	+	-	-	-	-	+	-	+
Offset Plane Wave	-	-	x	+	-	x	-	+	-
Common azimuth	+	-	+	+	-	+	+	+	+
Narrow azimuth	+	-	+	x	-	+	+	+	+

- poor
x adequate
+ best attainable

Figure 11. A comparison of 3D migration methods.

iment, diffractor targets, and flat reflector targets. Steep dips are better imaged in the DSR result, and runtime for DSR on 3D data is typically orders of magnitude faster than SPM.

Considerations when choosing a migration method. As discussed above, many choices must be made in selecting an imaging method, but the state-of-the-art in the industry is such that certain essential features should be offered in any choice of methods. When selecting wave-equation migration, all the following features should be verified (and the explorationist should be satisfied that the degree to which they are included is adequate to meet the desired exploration objective):

- proper preprocessing regularization (such as AMO)
- correct amplitudes
- high order extrapolation
- accurate handling of lateral velocity variations
- all recorded data included in the migration
- sufficient aperture in shot profile to capture steep dips

The key to depth imaging is the velocity model, and to get the correct velocity model, it is critical to be able to output prestack gathers so that wave-equation migration velocity analysis can be performed with angle- or offset-domain image gathers. It is also critical to perform the velocity updating in a manner that is consistent with the migration engine that will be ultimately used for the final image, and to be able to perform as many iterations as are necessary.

Conclusions. Based on algorithmic considerations and imaging results, we conclude that there are areas of applicability for most of the different imaging formulations. As shown in Figure 11, Kirchhoff and shot-profile wave equation algorithms are well suited for land and ocean bottom data; common azimuth and narrow azimuth wave-equation migration is best for marine streamer data. Kirchhoff has advantages in target-oriented applications and can complement wave-equation methods to build preliminary velocity models. Nonetheless, the explorationist should be aware of the strengths and weaknesses of the various imaging methods, the approximations and assumptions that are invoked, and what effect these will have on the desired outcome.

Suggested reading. "Imaging complex geologic structure with single-arrival Kirchhoff prestack depth migration" by Audebert et al. (GEOPHYSICS, 1997). "Imaging complex structures with semi-recursive Kirchhoff migration" by Bevc (GEOPHYSICS, 1997). "3D prestack migration of common-azimuth data" by Biondi and Palacharla (GEOPHYSICS, 1996). "Azimuth moveout for 3D prestack imaging" by Biondi et al. (GEOPHYSICS, 1998). "Stable wide-angle Fourier-finite difference downward extrapolation of 3D wavefields" by Biondi (SEG 2001 *Expanded Abstracts*). "Imaging a rugose salt body in the deep Gulf of Mexico: Kirchhoff versus common azimuth wave-equation migration" by Flieder et al. (SEG 2002 *Expanded Abstracts*). "Prestack Gaussian-beam depth migration" by Hill (GEOPHYSICS, 2001). "Common angle imaging with offset plane waves" by Mosher et al. (SEG 1997 *Expanded Abstracts*). "Prestack migration by split-step DSR" by Popovici (GEOPHYSICS, 1996). "Angle-domain common image gathers by wave-equation migration" by Prucha et al. (SEG 1999 *Expanded Abstracts*). "Fourier FD migration: The missing link between phase-shift and FD migration" by Ruhl and Ristow (SEG 1995 *Expanded Abstracts*). TLE

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