

## Least-square Attenuation of Reverse Time Migration Artifacts

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### SUMMARY

Reverse time migration artifacts occur when diving waves, head-waves or backscattered waves cross-correlate. These events are particularly strong where high velocity contrasts exist. Simple-minded filtering of the final image can lead to good results, but might compromise the integrity of the signal of interest. A better technique consists in applying a least-square filtering with prediction-error filters, a method traditionally used for signal/noise separation.

### INTRODUCTION

Reverse time migration (RTM) has spurred much interest in recent years due to (1) the increased imaging challenges posed by complex subsurface targets, and (2) affordable computer resources such as Linux clusters. RTM consists in solving the two-way wave equation for both the source wavefield and receiver wavefield with, for instance, a finite-difference scheme (Baysal et al., 1983; Whitmore, 1983) followed by an imaging condition (Claerbout, 1985). The imaging condition is a simple multiplication of the receiver and source wavefields (i.e., zero-lag cross-correlation), which are then summed over time and shot position.

RTM methods have the ability to migrate any type of multiples (surface and internal) to their correct location in the subsurface (Youn and Zhou, 2001), can handle multi-pathing, image turning waves and handle steep dips. All these properties are intimately linked to our ability to derive a very accurate velocity model. In addition to its sensitivity to the velocity model, RTM is also known for producing low-frequency artifacts where sharp velocity contrasts are present. These artifacts are well understood: RTM corresponds to the first iteration of waveform inversion (Tarantola, 1984) which produces a gradient vector to update the velocity model. In practice, these artifacts are created by the unwanted cross-correlation of headwaves, diving and back-scattered waves at the imaging step.

There are different alternatives for attenuating these artifacts. We classify these techniques into three categories:

1. Wavefield propagation approaches, where the wave equation is modified to attenuate reflections at the boundaries (Baysal et al., 1984; Fletcher et al., 2005).
2. Imaging condition approaches, where only the energy created by reflections is kept in the final image (Valenciano and Biondi, 2003; Yoon et al., 2004).
3. Post-imaging condition approaches, where the artifacts are filtered after imaging on each shot or the stacked image (Youn and Zhou, 2001). The filtering can be done on subsurface offset or angle gathers, as well (Sava and Fomel, 2005; Biondi and Shan, 2002).

We will not describe each of these techniques. Our experiments seem to indicate, however, that the method of Poynting vectors advocated by Yoon et al. (2004), although working well for simple model, does not produce satisfactory results in complex subsurface. We attribute this to the fact that at any given time and any given location in the subsurface, because of complex wave propagation,

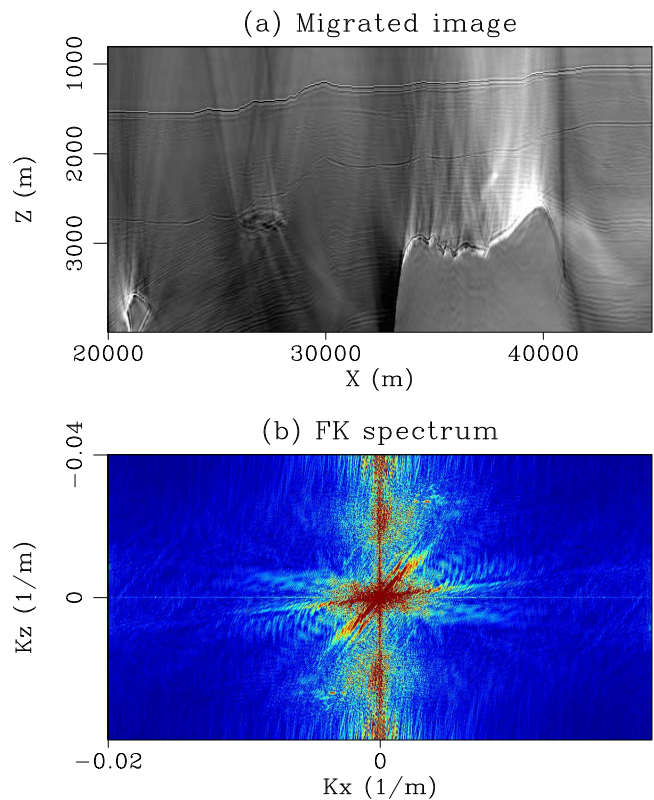


Figure 1: (a) RTM result of the BP dataset when no filtering is applied with (b) its 2D spectrum. The water bottom corresponds to the first reflection above  $Z=2000$  m. The complex salt body between  $X=32000$  m and  $X=40000$  m is well imaged. The migration artifacts form a low-frequency veil that masks the reflections.

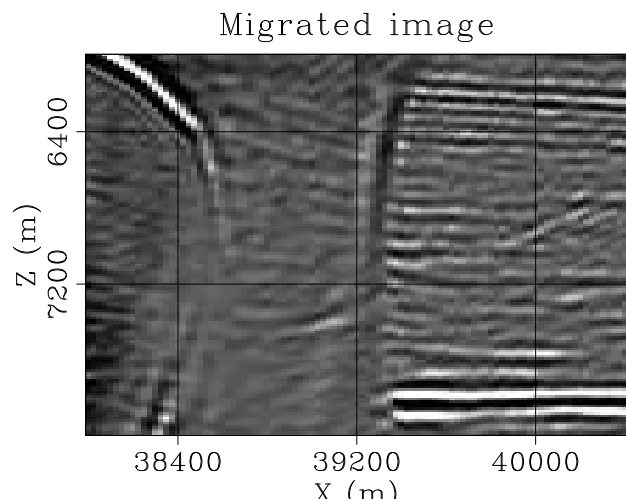


Figure 2: Close-up on the salt flanks for the RTM result with artifacts. Note that these flanks are very well imaged.

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there are many events going in different directions. This makes the extraction of one vector for the receiver and source wavefields difficult to achieve. It is also well-known that the non-reflective wave equation of Baysal et al. (1984) does not perform well for high angles of incidence.

In this paper, we investigate the third approach, for its simplicity and fast execution. First, we present simple filtering approaches that can be used for removing the artifacts. These filtering methods have the drawbacks of changing the phase and spectrum of the final image, and can attenuate useful information. Second, we advocate an inversion procedure that preserves the phase and the signal integrity while attenuating most of the artifacts. To achieve this goal, we use the framework of signal/noise separation theory (Guitton, 2005). We illustrate these techniques on the BP dataset (Billette and Brandsberg-Dhal, 2005) and demonstrate that the migration artifacts can be attenuated.

### FILTERING OF IMAGING ARTIFACTS

In this section, we show different filters applied to the RTM result. First, we show in Figure 1 the result of RTM with the artifacts and its corresponding 2D spectrum. The image is contaminated with a very low frequency veil throughout. These artifacts are particularly strong where the salt body is present between  $X=30000\text{m}$  and  $X=40000\text{m}$ .

#### Derivative filtering

Because the artifacts create a zero frequency component, it is natural to first try a derivative in the vertical or horizontal direction. We choose a vertical derivative of the form  $(1, -1)'$ , where  $'$  is the transpose. The results are shown in Figure 3. The image is clean but this filtering has three major effects.

1. It rotates the phase.
2. It removes vertical events.
3. It increases the high frequencies, where the noise resides.

The filtering effect on some reflections can be seen in the FK plot of Figure 3 where all the vertical energy is removed but also in Figure 4, where comparing with Figure 2, we see the complete removal of the salt flank at  $X=39200\text{m}$ . The artificial increasing of high frequencies in the final image can be solved by using a 2nd order derivative of the form  $(1, 0, -1)'$ , but it won't preserve vertical reflections, however. Although not shown here, a better alternative to the derivative is the Helix derivative of Claerbout (1998). The Helix derivative is obtained by factorizing the Laplacian operator into minimum phase filters. This filter won't affect the vertical or horizontal dips. It will, however, change the phase of the reflections.

#### Laplacian filtering

The second simple filter that we can use is a 2D Laplacian operator (Youn and Zhou, 2001). The result with a Laplacian filter in Figure 5 shows a good attenuation of the migration artifacts with two major effects:

1. It increases the high frequency noise.
2. It removes the low frequency information.

However, the vertical dips are preserved, as shown in Figure 6.

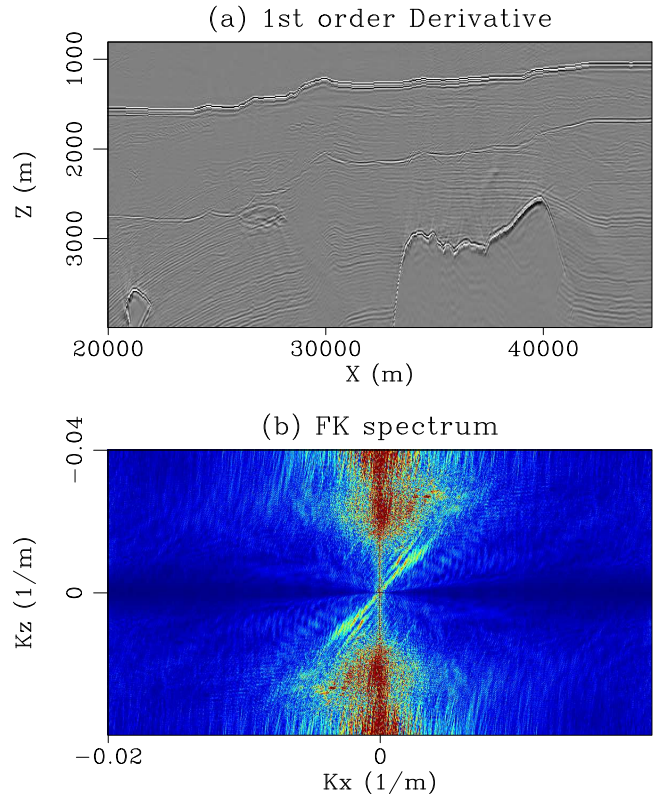


Figure 3: (a) RTM result with 1st order vertical derivative filtering and (b) its 2D spectrum. The steep salt flanks at  $X=32000\text{m}$  are attenuated and the phase of each reflection has rotated. This attenuation is visible in the FK plot as a zeroing of the energy at  $Kz=0$ . Note also the increase of energy for the high vertical wavenumbers in the FK plot.

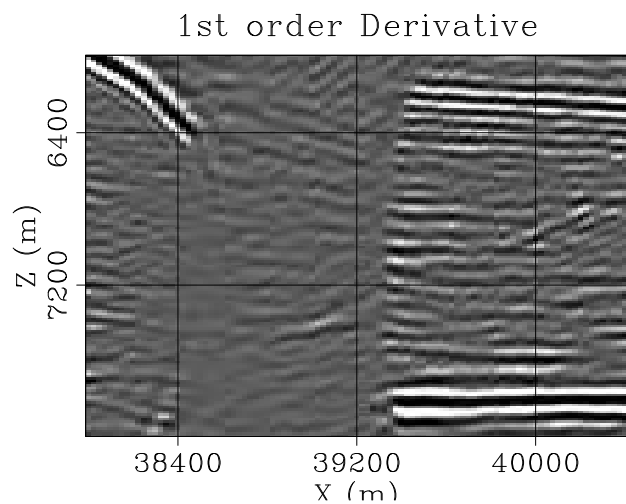


Figure 4: Close-up after 1st order derivative is applied: the flanks at  $X=39200\text{m}$  are gone and the high frequencies are artificially enhanced. Look at Figure 2 for comparison.

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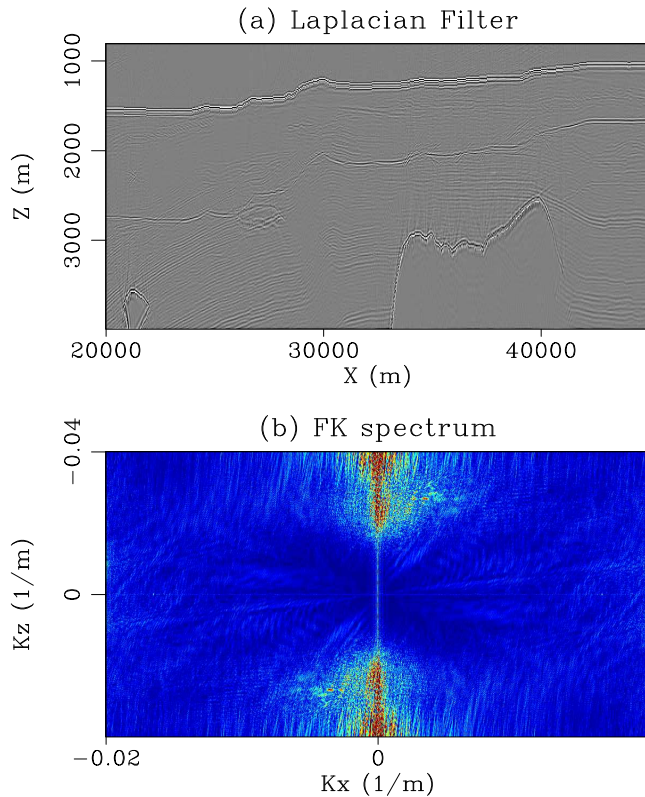


Figure 5: (a) RTM result with 2D Laplacian filtering and (b) its 2D spectrum. The low frequency information is attenuated, leaving high frequency noise. The salt flanks are preserved. The FK plot show the strong attenuation of the low frequency components at  $Kz=0$  and  $Kx=0$ .

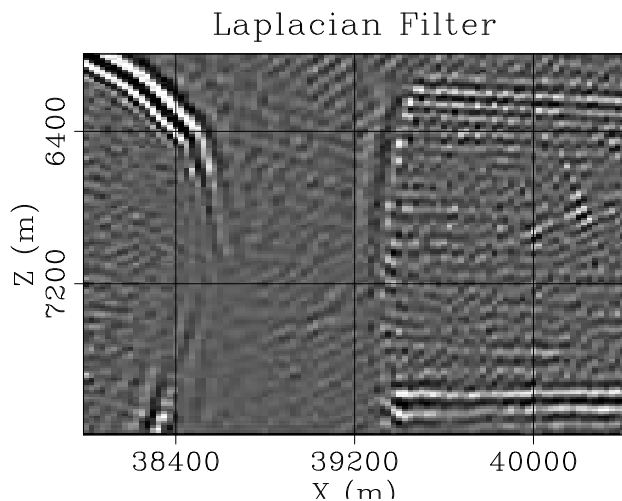


Figure 6: Close-up after a Laplacian filter is applied: the flank at  $X=39200m$  is preserved. The noise, seen as a salt and pepper pattern, is enhanced. Look at Figure 2 for comparison.

The main problem with a simple filtering is that it changes the character of the reflections by either rotating or filtering them. A desirable filter is a filter that removes the artifacts while preserving the integrity of the reflections. What we want is a projection filter (Soubaras, 1994).

### LEAST-SQUARE FILTERING OF IMAGING ARTIFACTS

Least-square filtering has been successfully applied for random noise (Soubaras, 1994) or coherent noise attenuation (Guitton, 2005). The idea is to treat the problem of removing RTM artifacts as a signal/noise separation problem in order to preserve the reflections as well as possible.

First, let's call  $\mathbf{A}$  an artifact annihilation filter and  $\mathbf{R}$  a reflection annihilation filter. We can define the RTM image  $\mathbf{m}$  as follows:

$$\mathbf{m} = \mathbf{m}_r + \mathbf{m}_a, \quad (1)$$

where  $\mathbf{m}_r$  is the image with reflections only (what we want) and  $\mathbf{m}_a$  are the artifacts (the noise). We introduce two residual vectors  $\mathbf{r}_a$  and  $\mathbf{r}_r$  such that:

$$\begin{aligned} \mathbf{0} &\approx \mathbf{r}_a = \mathbf{A}(\mathbf{m} - \mathbf{m}_r), \\ \mathbf{0} &\approx \epsilon \mathbf{r}_r = \epsilon \mathbf{R} \mathbf{m}_r, \end{aligned} \quad (2)$$

where  $\epsilon$  is related to the signal/noise ratio. The goal is to find  $\mathbf{m}_r$  in a least-squares sense:

$$f(\mathbf{m}_r) = \|\mathbf{r}_a\|^2 + \epsilon^2 \|\mathbf{r}_r\|^2. \quad (3)$$

The least-squares inverse is given by

$$\hat{\mathbf{m}}_r = (\mathbf{A}'\mathbf{A} + \epsilon^2 \mathbf{R}'\mathbf{R})^{-1} \mathbf{A}'\mathbf{A} \mathbf{m}. \quad (4)$$

In this paper, we choose prediction-error filters for both  $\mathbf{A}$  and  $\mathbf{R}$ . Thanks to the helical boundary conditions (Claerbout, 1998), these filters can have any dimension.

### Least-square PEF filtering

We first need to estimate a PEF for the artifacts and a PEF for the reflections. In this case, because the signal/noise ratio is so low in the regions where artifacts are present, we decided to keep  $\mathbf{R}$  equal to the identity operator and set  $\epsilon$  very close to zero (0.01). This amounts to solving

$$\hat{\mathbf{m}}_r = (\mathbf{A}'\mathbf{A} + \epsilon^2 \mathbf{I})^{-1} \mathbf{A}'\mathbf{A} \mathbf{m}, \quad (5)$$

which shows that we are essentially deconvolving the PEF plus a prewhitening factor  $\epsilon$  (Soubaras, 1994). For the noise PEF  $\mathbf{A}$ , we estimated it from the artifacts present in the water column only. The result of least-square filtering are shown in Figures 7 and 8. As seen in the FK spectrum in Figure 7b, the reflections are better preserved than with the filtering techniques. Plus, there is no phase rotation of the signal. Figure 8 illustrates that the steep dips are also well preserved.

### Least-square filtering before or after shot stacking

In the results presented above, the filtering (least-square or not) is done after all the shots were stacked. If the same filter is used for all the shots, e.g. a derivative, it does not matter if we filter before or after the summation. At the contrary, for the least-square filtering procedure, better results would be obtained by estimating the PEFs for the artifacts and the reflections on each shot, and separating the noise and signal on each shot. This can be easily automated during the migration process with a negligible impact on the total run time.

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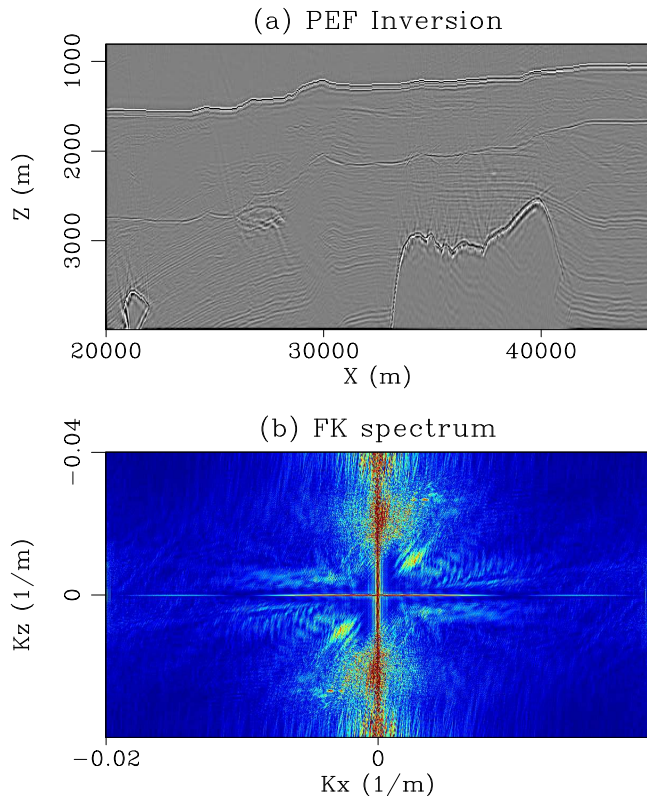


Figure 7: (a) RTM result with least-squares filtering and (b) its 2D spectrum. The reflections are preserved. Some migration artifacts are still present above the top of salt at  $Z=3000\text{m}$ . This might be improved by using non-stationary filters and doing the least-square filtering before stacking the shots. Comparing the FK plot with Figure 1b, we see that the spectrum is well preserved except where the artifacts are present, i.e., at  $Kz=0$  and  $Kx=0$ .

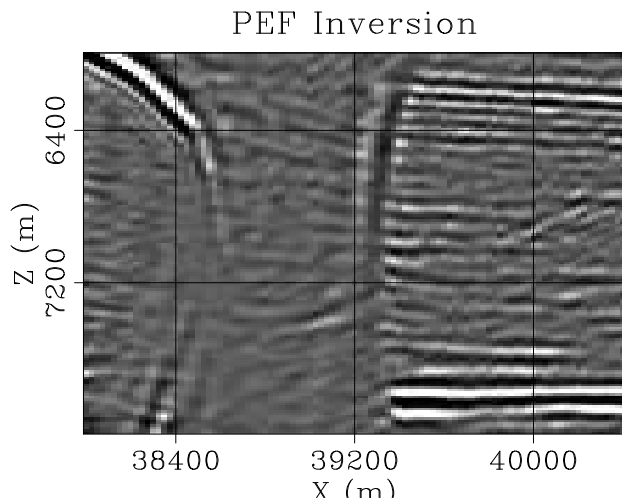


Figure 8: Close-up after least-squares filtering with a PEF. Compared to Figure 2, the artifacts are attenuated. Compared to Figures 4 and 6, the steep dips are preserved and the high frequency noise is not enhanced. Also, there is no phase difference with the input image.

### DISCUSSION

We presented a technique to attenuate RTM artifacts after imaging condition. This method operates a least-square filtering of the imaging artifacts with prediction-error filters. Here we demonstrated stationary filters only. Better attenuation results can be possibly obtained by estimating local filters for both the artifacts and the reflections, similar to what we do for coherent noise attenuation (Guitton, 2005). In addition, replacing the identity operator  $\mathbf{I}$  with genuine signal annihilation filters improves the attenuation result. Other sources of improvement can come by applying the filtering on subsurface offset or angle gathers, before stack. In any case, any attempt at removing RTM artifacts should be done in such a way that all the reflections are kept intact.

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