

# Wave Path Tomography Provides 'New Tech' Solution For Seismic Velocity Modeling

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SANTA CLARA, CA.—In areas of complex velocity that are challenging for ray-based velocity inversion methods, wave path tomography offers a new alternative that is consistent with the wave equation migration methods used to produce the seismic image.

The goal of wave path tomography is to create a method for tomographic velocity updating that combines the speed and robustness of ray-based travel time tomography using a back projection operator derived from the migration algorithms based on the wave equation. A wave path is obtained by multiplying impulse responses from a surface location and a reflection point. Wave path (or sensitivity) kernels have been used in numerous applications, particularly for velocity model updating based on transmitted (refracted) seismic waves or electromagnetic waves. Now the wave path concept is being applied to reflection tomography problems to build velocity models in a way that is consistent with the wave field migration.

The best known method of velocity model updating using wave path back projection is wave form inversion. Wave form inversion minimizes the difference between a modeled wave field and the recorded wave field. The modeling operator is the same as in reverse-time migration: a two-way implementation of the (acoustic or elastic) wave equation. This is a nonlinear minimization problem that is solved by a steepest descent technique. Its solution can easily fall into a local minimum.

Therefore, the starting velocity model should be close to the expected solution. Rather than estimating the scattered wave field and matching it with the observed wave field, a more modest approach that requires less computation and is applicable in 3-D characterizes the mismatches between trial and actual velocity by residual travel times that are measured from the common image gathers, and back propagates these travel times along the wave paths (instead of the ray paths).

Obviously, an essential part of this technique is to derive accurate relationships between the residual reflection travel times and the wave paths. By relying on the back projection of residual travel times rather than image perturbations or waveform matching, some of the advantages of a wave field-based back projection operator are retained while avoiding much of the computational demands and complexity of other methods. Furthermore, travel time inversion is more robust regarding the quality of the starting velocity model.

### Three Unknowns

The reflection tomography problem requires three unknown pieces of information in addition to the known acquisition geometry of surface source and receiver locations. The three unknowns are:

- The back projection operator derived from an assumed slowness model;
- The location and orientation of subsurface reflectors; and
- A measurement of the deviation of modeled from recorded travel time.

travel time.

To be viable as a velocity updating method for large seismic surveys, all three components need to be found in an automated way.

The automatic selection of reflection points for the back projection operator (or back projection “points”) is based on the geological dip and reflector continuity information that can be extracted from the stack of the migrated seismic image. The goal is to restrict velocity updating to the best data in the prestack image, employing a “limited picking” algorithm. Back projection points are based on reliability factors extracted from the stack volume, rather than picked reflectors.

The first step is to calculate the local reflector slope at each point of the stacked migrated image (dip field) and the coherency of this dip estimate. A recursive search then selects likely reflection points based on specified levels of signal amplitude, dip coherency, and spatial density (minimum distance between points). In each iteration of the volume search, these criteria are relaxed to find the best points in each region. The first pass accepts points above a high threshold, such as the 90th percentile of amplitude and coherency, while each subsequent pass lowers this threshold to fill in regions without picks from the previous passes. Points that fall into regions that should be excluded from the velocity update (such as a water layer or salt bodies) are discarded.

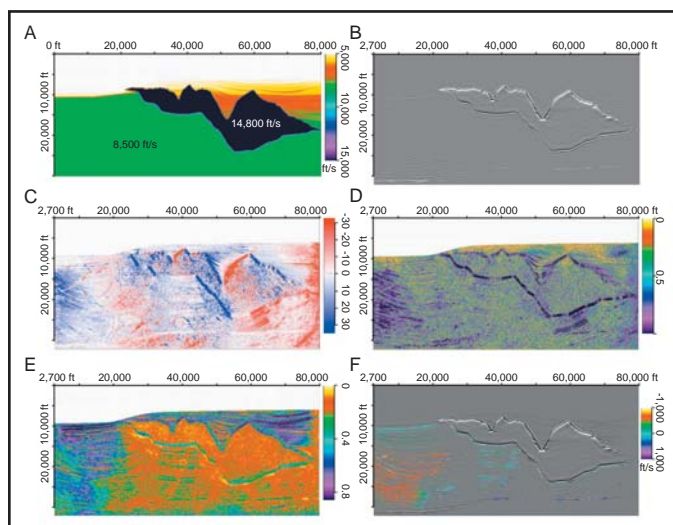
The calculated dip field at the selected locations defines the reflector “normal,” and therefore, the initial direction of the back projection operator—be it a ray or a wave path. The application of this operator to Sigsbee synthetic data is illustrated in Figure 1. The six panels show the velocity model (A) used to generate the common azimuth depth-migrated seismic image (B), the dip field (C), dip coherency (D), semblance strength of residual velocity picks (E), and selected reflection points (F) displayed as dip bars with color-coded residual velocity.

From here, conventional ray-based travel time tomography proceeds just as if complete horizons had been picked instead of isolated back projection points. A horizon can be thought of as a collection of back projection points whose normal direction is determined by the horizon geometry, rather than directly from the local dip direction of the image.

If the picked horizon exactly follows the image dip, both ways of generating back projection points yield identical results. The residual travel time data for the tomographic inversion can be derived directly from the residual depth error observed on the prestack common image gathers (residual move-out analysis). This type of tomography often is referred to as “multiparameter” inversion. An alternative method uses semblance analysis (velocity spectra) to quantify the residual move-out with a single value by fitting hyperbolic move-out trajectories. This “single-parameter” inversion is analogous to the stacking velocity analysis of time-domain processing, and has been adapted to different imaging domains.

The semblance value at a given reflection point gives yet another criterion to select the best back projection points. Points that fall below a specified threshold of semblance strength, as well as points that are associated with semblance picks outside a specified range of velocity residuals to avoid outliers (such as residual velocities from multiples) are discarded.

**FIGURE 1**  
**Reflection Point Selection for**  
**Subsalt Velocity Model Updating (Sigsbee Model)**



## Wave Path Propagation

With reflector locations and the migration velocity model, the back projection operator can be derived. In ray tomography, this is achieved by shooting rays from the reflector to the surface over a given range of opening angles around the reflector normal. In wave path tomography, the propagation direction is unknown and must be determined by trial and error while building the wave paths.

The wave path is the scalar product of impulse responses of the seismic propagator (migration operator) to the velocity model at a specific frequency. The sources of the impulses are the two “end points” of the wave path—in this case, a surface source location and a subsurface reflection point. By integrating (summing) over a range of frequencies in the seismic frequency band, monochromatic wave paths can be replaced with band-limited wave paths in the time domain, although single-frequency (monochromatic) wave paths are typically sufficient.

The raw wave paths extend over the entire velocity model, or over whatever aperture one chooses to calculate the wave fields. This implies a very large inversion problem, which may not be solvable in practice for large data sets. The shape of the wave path depends on the frequency (or range of frequencies) that is propagated. The lower the frequency, the wider the zone of significant energy around the theoretical ray path. The amplitudes fall exponentially away from the ray path. To manage the size of the resulting inversion matrix, the wave paths are restricted to the region where the propagated energy is concentrated by muting the small contributions from the far field.

One way to delineate a mute function is to choose the zero crossings of the imaginary part of the complex-valued wave path using an event tracking technique. The same technique is used to accelerate and stabilize the search for “center rays” in the wave paths. In the case of normal-incidence propagation from coincident surface sources and receivers, we now have our final wave path. Oblique incidence with an offset between a source and receiver results in a wave path with three “corners:” the two true end points at the surface, and the subsurface reflection point. An exact calculation of this wave path would require a two-way propagator, but a one-way propagator can approximate this case as a superposition of two wave paths emanating from the reflection point (“exploding reflector” model) to the source and receiver positions.

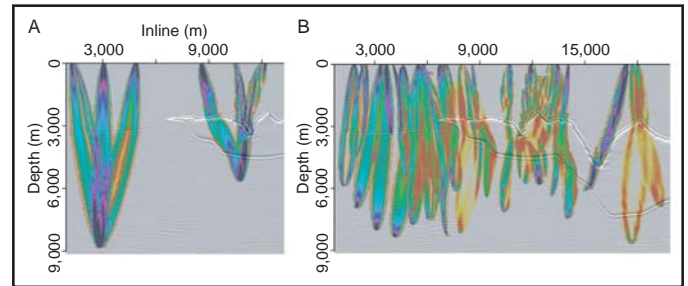
Examples of muted wave paths are illustrated in Figure 2 on the synthetic 2-D Sigsbee model. Panel A shows wave path fans (normal-incidence and offset wave paths from the same reflection point) in the sedimentary section and through the salt body, while Panel B shows normal-incidence wave paths with color values proportional to the inversion matrix coefficients.

## Constructing Wave Paths

To construct the wave paths from a set of selected reflection points and their associated up-going wave fields, surface points must be found that correspond to propagation at normal incidence from the reflectors to compute the down-going wave fields. Since it is not known in advance where these surface locations are, the down-going wave fields are precomputed from a grid of surface locations, and each reflection point is then searched for

FIGURE 2

### Sample Wave Paths from 2-D Sigsbee Model



the best match. The simplest method is to calculate the dip of the down-going wave field at the reflection point and match it with the reflector tangent. Alternatively, the amplitude maximum of the product of up-going and down-going wave fields (the trial wave path) can be tracked up from the reflection point to find the best fit.

In ray tomography, a straightforward relationship between velocity error and travel time or depth residuals can be derived. The analogous relationship for wave path tomography based on travel time differentials between modeled and recorded wave forms is far more complex, but the linearized residual travel time equation of conventional ray-based tomography can be retained for wave path tomography without using ray properties or ray tracing.

The last piece of the procedure is measuring residual travel times (the data vector of the tomographic inversion). A widely used and robust method is to derive the travel time residual from a single-parameter hyperbolic curve fit to the observed residual move-out in the common image gathers (semblance analysis). In cases of small-scale heterogeneities, this approach may not yield the required resolution and a multiparameter inversion based on the actual residual move-out (RMO) values from individual image traces becomes desirable.

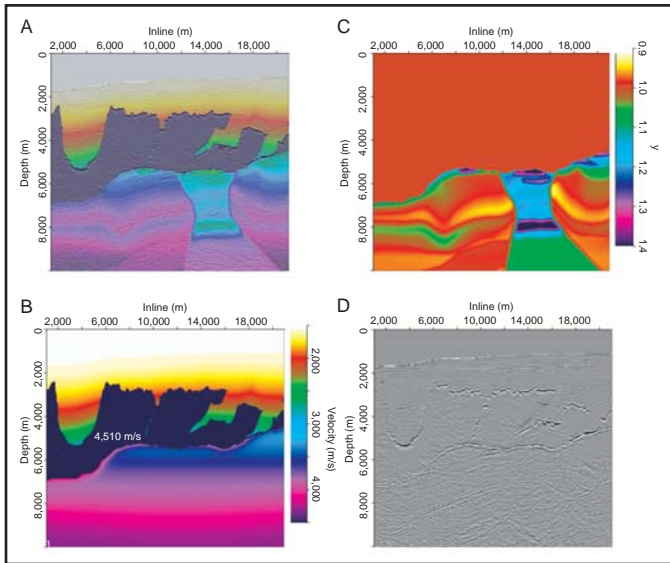
Picking individual RMO values from a large prestack image is only feasible with an automatic procedure. Automatic residual move-out picking is based on a “flattening” method in which the events to be flattened are reflections in the prestack domain of common image gathers that show residual move-out caused by errors in the migration velocity model. The picking procedure involves three steps: calculating the dip field for each common image gather, integrating the dip field to find relative depth shifts, and extracting the depth shifts at the desired normal incidence locations.

## Synthetic Text Case

As a test case, these three techniques were combined on a widely used 2-D synthetic data set provided by BP. The goal was to detect the narrow, low-velocity anomaly below the left salt body, which represents a drilling hazard. For the starting model, it was assumed that the velocity above salt and the salt bodies was known exactly, while the velocity below salt was a simple gradient extended laterally from the center of the section. The ratio between starting and true velocity ranges from 0.76 to 1.57.

Panel A in Figure 3 shows the true velocity model overlaid on a common-azimuth depth-migrated image of the BP synthetic

**FIGURE 3**  
**BP Synthetic Seismic Model Text Case**

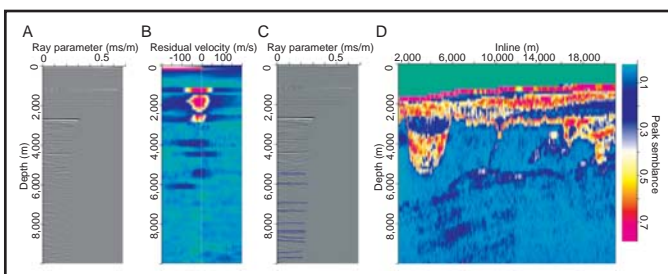


seismic model. Panel B is the starting velocity model for subsalt migration velocity analysis of the synthetic seismic data, and Panel C is the ratio between starting and true velocities. Panel D is the depth-migrated image of the synthetic seismic data using the starting velocity model.

The synthetic data was migrated with a common-azimuth algorithm using a phase-shift downward continuation operator. Reflector continuity in the zone of interest was very poor, which also was reflected in the low semblance strength of the common image gathers below salt (Figure 4). The four panels show the common image gather (offset ray parameter), semblance gather (residual velocity), residual move-out picks from the depth-migrated image, and the semblance strength map.

Although the semblance peaks of the stronger subsalt reflectors are well focused, velocity updating based on semblance-derived residual velocities is unlikely to work well in this case because of the assumptions of single-value inversion (hyperbolic residual move-out over common image gathers aperture), which do not hold for such strong lateral velocity gradients and complicated overburden. By using the depth error (residual move-out) of reflectors on individual traces of the common image gathers, the limitations of single-value residual velocity inversion can be overcome.

**FIGURE 4**  
**Reflector Continuity in Zone of Interest**



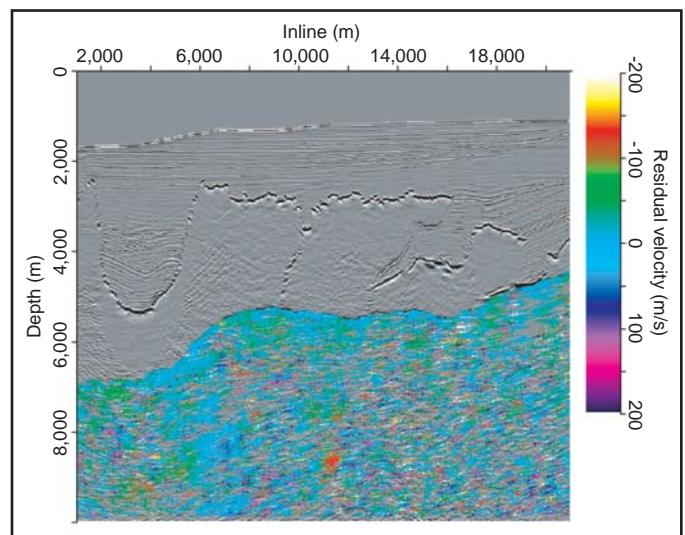
Starting from the dip fields of stack and common image gathers, automatic picks of back projection points, reflector normal directions and residual move-out were generated (Figure 5). A plane-wave destruction filter was used to calculate the dip field, since the speed and robustness of this algorithm make it especially attractive for processing potentially large prestack data sets.

The back projection data are fed both into a ray-based and a wave path-based tomographic workflow. The inversion contains 22,000 back projection points and the back projection paths cover an offset range of 15 kilometers. Because of the lack of coherent reflectivity in the (to be resolved) subsalt low-velocity zone, relatively fewer back projection paths actually originate within it. Setting the quality criteria for accepting back projection points low (semblance strength is not used to reject points) ensures even and dense coverage. This, of course, entails the danger of accepting residual picks of dubious quality, but the redundancy achieved with the large number of back projections should mitigate this problem.

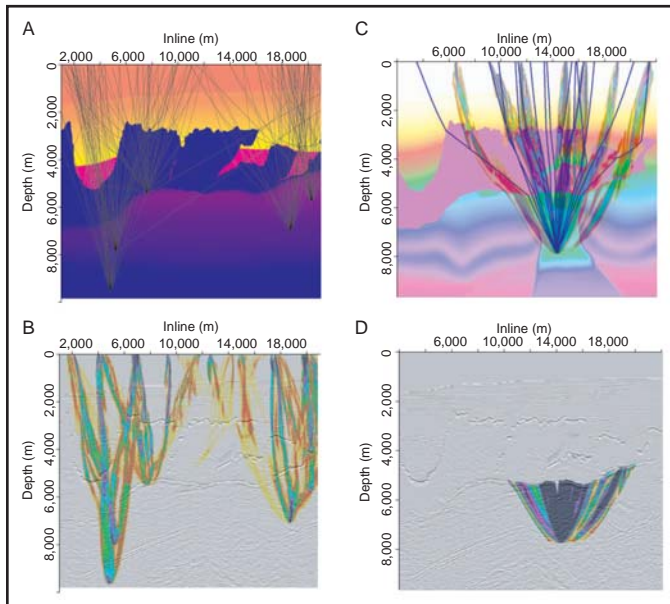
The ray fans consist of the normal incidence ray and up to 10 offset source-reflector-receiver rays that are calculated in constant opening angle increments up to the maximum surface offset (not every ray can be successfully traced to the surface). The wave paths are calculated at 10 Hertz (average wavelength 372 meters) with a surface grid spacing of 250 meters and an offset sampling of 2,500 meters. The velocity model (and wave fields) are sampled at 12.5 meters vertically and 25 meters horizontally. Figure 6 shows sample ray path (Panel A) and wave path (Panel B) fans from 11 reflection points (every 2,000th reflection point in Figure 5). Generally, rays and wave paths follow very similar trajectories, although rays are occasionally deflected wildly (including multiple reflections) by the complicated top salt interface. Since the velocity model is known to be correct above the salt base, only the subsalt parts of the back projection paths contribute to the back projection matrix.

Panel C in Figure 6 shows the ray and wave path fans from a

**FIGURE 5**  
**Selected Reflection Points (Displayed as Dip Bars with Color-Coded Residual Velocity)**



**FIGURE 6**  
**Sample Ray and Wave Path Fans**



reflection point (dip bar) in the subsalt low-velocity zone overlaid on the true velocity model, while the fourth panel shows the wave path fan after restricting to the subsalt part of the velocity model overlaid on the stacked image. Note that the low-velocity zone shows very little coherent reflectivity in the migrated image.

The inversion matrix generated by the wave path method is approximately seven times denser (number of non-zero entries) than the ray-based matrix. A gradient solver with multiscale regularization was conjugated to compute velocity updates.

### Low-Velocity Zone

While both wave path and ray tomography methods picked up lowered velocities below the salt base, only the wave path inversion came close to the true magnitude near the salt base and to the fact that there was a low-velocity zone extending to greater depth. The location of the deeper low-velocity zone was not well distinguished, but the general trends of the subsalt velocity structure on both sides of the low-velocity zone start to appear.

Figure 7 shows the remigrated images using updated velocity models from ray path (Panel A) and wave path (Panel B) tomography. Remigrating the data with the wave path-updated velocity model improved subsalt reflector continuity. By contrast, the changes in the ray-based velocity update are so slight that the migrated image does not improve visibly. Further iterations could potentially improve the velocity resolution, but the remaining traceable residual move-out proved to be too small to achieve significant improvements with the given common-azimuth migration method.

Algorithms such as shot profiling or reverse-time migration methods have been shown to improve subsalt imaging in the BP model. Therefore, the potential exists for further progress with travel time velocity model building methods before resorting to wave form-based inversions. Alternatively, focusing analysis,

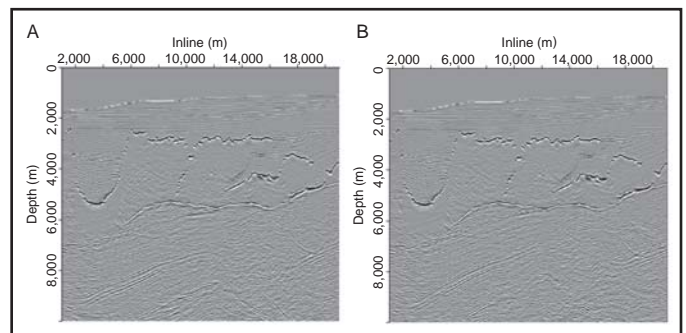
such as constant velocity stacking, might be able to determine residual velocities in the areas of poor reflectivity such as the subsalt low-velocity zone that the move-out analysis was able to detect, but not resolve in detail.

The advantage of tying wave path tomography to the simplicity of travel time inversion also has its limitation. Although the details of a traced ray are not needed to derive the inversion matrix (requiring only a bulk property such as ray length or total travel time), it still uses only the kinetic (move-out) information from the wave field to derive the inversion data vector. To include amplitude information, an approach such as differential residual migration, which combines methods of differential semblance optimization and wave-equation migration velocity analysis, could be explored.

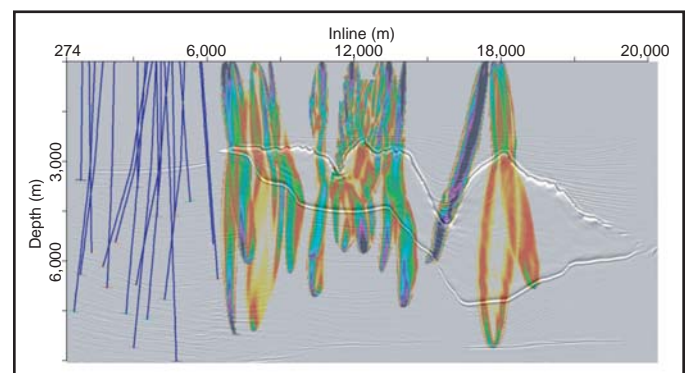
As this example shows, the wave path tomography method yields results superior to ray tracing in areas of complex overburden. The ability to resolve the low velocity overpressure zone below the complex salt body illustrates the utility of wave path tomography for not only velocity model building, but also for hazard detection.

In practice, wave path tomography will be reserved for the parts of the velocity model that cannot be obtained by ray tomography. In fact, the two approaches are easy to combine. For example, in the Sigsbee case, the sedimentary velocities outside the salt body can be easily obtained from ray tomography. A

**FIGURE 7**  
**Remigrated Images Using Updated Velocity Models**



**FIGURE 8**  
**Joint Ray and Wave Path Back Projections (Sigsbee Model)**



combined inversion is illustrated in Figure 8, with selected normal rays and wave paths indicating the contribution of each.

Even where the velocity field is very smooth and simple, as in the Sigsbee case, wave path tomography adds robustness to the inversion. Wave path tomography avoids problems with erratic ray paths resulting from multiple internal or “underside” reflections generated by the high velocity contrast between the sediments and the salt body, or strong deviations of the ray path caused by a rugose salt boundary. □

**Editor’s Note:** For more information on wave path tomography technology, see “Automated Velocity-Model Building with Wave Path Tomography,” a technical paper the co-authors published in the September-October 2008 issue of *Geophysics*, the Society of Exploration Geophysicists’ archival journal.

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